Combining the midlatitudinal and equatorial background error in global data assimilation

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Overview

• Problem: Global data assimilation contains no tropical mass/wind relationship
• Methods: Background error in data assimilation via Hough modes
• Results:
  – Response to single observations
  – Effect of ADM-data in global data assim.
• Summary
The need of tropical wind data

Standard deviation of 48-hour forecast errors in the zonal wind field (m s\(^{-1}\)) of the ECMWF model at 200 hPa for September 2005.

Line-of-sight wind by ADM-Aeolus
Variational data assimilation

\[ J(\delta x) = \delta x^T B^{-1} \delta x + (y - H[x_b + \delta x])^T R^{-1} (y - H[x_b + \delta x]) \]

**Background:** \[ \delta x : \text{Increment} \quad x_b : \text{Background} \quad y : \text{Observations} \]

**Observational error:** \( R : \text{Observation error covariance} \)

**Background error covariance:** \( B : \text{Background error covariance} \)

**Forward model:** \( H : \text{Forward model} \)

This study: global shallow water model (2d)
Midlatitudinal and equatorial Hough modes

Rossby mode, $n=1$, $m=3$, $H=10^4$ m

Kelvin mode, $m=3$, $H=25$ m

Hough modes with zonal wave number $m=3$. Shadings show geopotential height (m), vectors show winds (m/s).

Problem: Modes not orthogonal. Vertical structure function?
A priori knowledge of the background errors (B)

• **Midlatitudes:**
  - Geostrophy
  - Rossby waves
  - Mixed Rossby-gravity w.
  
  $\rightarrow$ Hough modes with equivalent depth $= 10,000$ m

• **Equator:**
  - Kelvin waves
  - Equatorial Rossby waves
  - Mixed Rossby-gravity w.
  - "Slow" west- and eastward inertia-gravity w.
  
  $\rightarrow$ Hough modes with equivalent depth $= 25$ m

For the assimilation each mode is assigned a certain variance which is deduced eg. from forecast errors.

**FOR NOW:** Red spectrum for each mode is assumed.
Combining midlatitudinal and equatorial B

-> Now, the increment is split in two: \( \delta x = \delta x_m + \delta x_e \)

-> Minimize cost function via control variables:

Midlatitudinal
\[ \chi_m = L_m \delta x_m \]

Equatorial
\[ \chi_e = L_e \delta x_e \]

\( L_{m/e} = D_{m/e} M_{m/e} \) with \( D_{m/e} \) : Normalisation with standard deviation of each Hough mode

\( M_{m/e} \) : Projection on Hough modes

\[
J(\chi_m, \chi_e) = \chi_m^T \chi_m + \chi_e^T \chi_e +
+ (y - H[x_b + L_m^{-1} \chi_m + L_e^{-1} \chi_e])^T R^{-1} \ldots
\]

\[
\ldots (y - H[x_b + L_m^{-1} \chi_m + L_e^{-1} \chi_e])
\]
Single zonal wind observation in midlatitudes

Analysis increment to a single zonal wind observation in midlatitudes at 30°N 0°E. Shadings show geopotential height (m) with positive values red and negative blue. Arrows show wind (m/s).
Single zonal wind observation at the equator

Analysis increment to a single zonal wind observation in midlatitudes at 0°N 0°E. Shadings show geopotential height (m) with positive values red and negative blue. Arrows show wind (m/s).
Application to global data: Simulation experiments

1. Create **artificial data** set with known covariances for eq. and midlat. Modes.

2. **Reconstruct** background error covariances $\mathbf{B}$ (ie. standard deviation of each mode $\mathbf{D}_{m/e}$).
   
   Problem: Non-orthogonality of modes

3. Create **observations** with random errors.

4. **Data assimilation** experiments
   
   • Influence of ADM-wind data on analysis
   
   • Effect of ”wrong” or ”right” background error
Artificial data

Random data $x_r$, day 1
Reconstructing $B$

- Determine daily amplitude $a_{m/e}$ of modes by minimizing error:
  $$\left\| x_r - a_m M_m - a_e M_e \right\|^2$$

- Midlatitudinal modes
- Equatorial modes
Effect of ADM-wind data in the global perspective

Global Root Mean Square Error of
Zonal wind $u$

Geopotential $Z$

Data assimilation with
new full $B$
(analyt.:solid, reconstr.:circles)
Only midlat. $B$
(dashed)

Normalized RMS

Number of observations
Effect of ADM-wind data in the tropical perspective

Equatorial (30°S-30°N) Root Mean Square Error of Zonal wind $u$ and Geopotential $Z$

<table>
<thead>
<tr>
<th>Data assimilation with</th>
<th>Only ADM ($u$)</th>
<th>Only Temp ($Z$)</th>
<th>ADM + Temp ($u,Z$)</th>
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- New full B (analyt.: solid, reconstr.: circles)
- Only midlat. B (dashed)
Summary

- A combined background error formulation includes the equatorial waves following Žagar (2004) retaining the midlatitudinal approach.
- A minimization technique retrieves the background error covariances for the non-orthogonal set of Hough modes.
- The ADM-Aeolus wind data will improve the global analysis with the new background error formulation, especially by reducing the tropical error.