

Space structures

5. Finite element procedures for static, dynamic, buckling and thermoelastic analyses.

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Mechanical engineering disciplines of interest for space applications

- CLASSIC MECHANICS (ASTRODYNAMICS)
- SOLID MECHANICS
- THERMAL CONDUCTION
- HEAT TRANSFER
- MOISTURE ABSORPTION
- NON REACTING FLUID MECHANICS
- REACTING FLUID MECHANICS
- COUPLED PROBLEMS (THERMOELASTICITY, F/S INTERACTIONS)
- ACOUSTICS

What problems are we interested in and what purpose is our model for

Analysis problems (the system and the input are known the output is to be determined and verified)

- Static response (stress field, failures, deformation)
- Time response
- Eigenfrequencies and eigenmodes
- Stability of equilibrium positions
- Stability of motion

Design problems (the input is known, the output is prescribed (completely or in part), the systems is to be determined, controlled or modified). As to design there are no unique solutions

- Optimisation
 - Control problems
 - Modification problems
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On the modeling of problems of classical mechanics

- In classical mechanics, a rather limited field of the human knowledge, many different mathematical models of the physical behavior of matter have been developed.
 - A short list might include: the single massive particle, systems of massive particles, the rigid body, systems of rigid bodies, the deformable 3D body, systems of rigid and deformable bodies (multi-bodies), the beam model, the shell model, the laminated shell model
 - The state of the above mentioned models are described either by discrete or continuous variables. Also the external actions and the characteristics of the system can be described by discrete or continuous functions.
 - The modeling of the physics could have the purpose of predicting the time response of the mechanical system to external actions, to evaluate the stability of this response, to better investigate the properties of the system or to design or modifying the system in order to obtain some desired response.
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Undeformable material systems

- For undeformable bodies the state of the system is described by discrete variables representing displacements, rigid rotations and the relevant derivatives w.r.t. time, the external actions are represented by scalars representing applied force, applied momentum, imposed displacement or rotations. The system is described by the Newton law of motion applied to every single particle or by the equations of stereodynamics for the rigid body case. As far as the constitutive material, the system need only to be characterized by the amount mass as a global parameter or a local one (density).
- The possible presence of a varying density from one point to another of a rigid body is the first use of entities that are variable in terms of space and not only in terms of time independent variables.
- An important aspect of the modeling of rigid bodies is the modeling of external forces (environment) that could have a complicated modeling in terms of the geometry of the body and depending upon the nature of the interaction between the body and the external world (e.g. aerodynamics).

Deformable material systems

- For deformable bodies two new class of state variables need to be established in order to describe the deformation and the internal stress.
- Internal stress are not excluded in the rigid body mechanics (that of course exclude the possibility of changing the mutual distance between two generic points of the body) but their presence has no consequence on the motion of the body.
- In a deformable body the constitutive particles are able to vary their mutual distances. A variable is needed for describing the presence of a change in shape occurred during a motion, that is a displacement of the massive parts of the system that cannot be described (or recovered) by a rigid body motion (translation and rotation). In most cases the deformation at a local level is of interest, not a global information of the system (since it is not easy to make in general a comparative study between two different deformed configurations of a material system).

Deformable material systems (3)

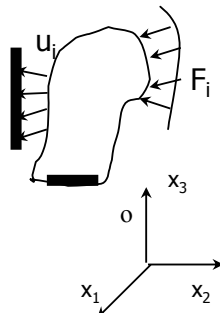
- In order to select (with some reliable criteria) possible functions that would be able to describe the deformation of the system at a local point and to exclude from this description rigid body motion a new description of the state variable of the system is needed.
- First the description of motion has to be extended to all the points of the system. A possible choice is to select the configuration of the system at one time as the reference and by labeling each material particle with the position in the reference configuration then to follow the motion of each of this particle in terms of their original position in the reference frame (Lagrangian approach): in this way the variables describing positions (and as a consequence velocities and acceleration) are continuous function not only of time but also of space independent variables (continuum mechanics), since a continuous distribution of massive points is assumed to be present.
- Then a relation between positions (or displacements) and the deformation variables has to be set up: compatibility equations (definition of deformation in terms of positions).
- In this way both a new class of variables and a new kind of relations (apart from the ones concerning equilibrium) have been introduced

Deformable material systems (4)

- It is part of the common experience the fact that different kind of materials subjected to the same external actions produce different deformations.
- In order to investigate this effect at a local level (as done in the study of deformation) a new set of state variables, the stress field, should be introduced.
- With this introduction it is possible to write equilibrium equations in a more general way, that is considering not only the entire body but parts of it. Internal actions are exchanged between the parts of the bodies according to the third law of Newtonian mechanics. The equilibrium relations establish the balance between external forces, inertia forces and the internal stress field.
- The balance between internal and external forces does not vary from one material to another.
- Indeed there is a change considering the effect of the stress field on deformation. This aspect is described by the constitutive relations that relate stress with deformation. The third class of relations is finally established along with the three kinds of state variables.

SOLID MECHANICS EQUATIONS

VARIABLES



EQUATIONS

STATE VARIABLES

Displacement	$u_i(x_i, t)$	$i=1,2,3$ $u(x,y,z,t), v(\dots), w(\dots)$
Strain	$\epsilon_{ij}(x_i, t)$	$\epsilon_{xz}, \epsilon_{yz}, \epsilon_{yz}, \epsilon_{xz}, \epsilon_{xy}$
Stress	$\sigma_{ij}(x_i, t)$	$\sigma_{xz}, \sigma_{yz}, \sigma_{yz}, \tau_{xz}, \tau_{xy}$

EXTERNAL ACTIONS on V, S_u, S_f

Body (volume) forces	$X_i(x_i, t)$	$X(x,y,z,t), Y(\dots), Z(\dots)$
Surface forces	$f_i(x_i, t)$	$f_x(x,y,z,t), f_y(\dots), f_z(\dots)$
Imposed displacement	$\hat{u}_i(x_i, t)$	$u(x,y,z,t), v(\dots), w(\dots)$

EQUILIBRIUM

$$\sigma_{ij,j} + X_i = 0 \quad \text{on } V$$

$$n_i \sigma_{ij} = f_i \quad \text{on } S_f$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + X = 0$$

COMPATIBILITY

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

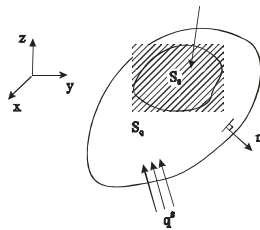
$$u_i = \hat{u}_i \quad \text{on } S_u$$

$$u_{i,j} = \frac{\partial u_i}{\partial x_j} \quad \text{repeated indices = summation} \quad u_{i,j} = \sum_i \frac{\partial u_i}{\partial x_j}$$

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

HEAT CONDUCTION EQUATIONS (1)

VARIABLES



EQUATIONS

STATE VARIABLES

Temperature	$T(x,y,z,t)$
Temp. gradient.	$dT/dx, dT/dy, dT/dz$
Heat flux	$q_x(x,y,z,t), q_y(x,y,z,t), q_z(x,y,z,t)$

EXTERNAL ACTIONS on V, S_u, S_f

Generated heat per unit volume	$q^B(x,y,z,t)$
Surface generated heat flux	$q_i(x,y,z,t)$
Imposed temperatures	$T(x,y,z,t)$

EQUILIBRIUM

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) = -q^B \quad \text{on } V$$

$$k_n \frac{\partial T}{\partial n} \Big|_{S_q} = q^S \quad \text{on } S_f$$

COSTITUTIVE

$$q_x = -k_x \frac{\partial T}{\partial x} \quad ; \quad q_y = -k_y \frac{\partial T}{\partial y} \quad ; \quad q_z = -k_z \frac{\partial T}{\partial z}$$

Boundary conditions

Thermal boundary conditions

Conditions on temperature (essential bc):

θ_S ▶ Imposed temperature on S_T

Conditions on heat flux (natural bc):

k_n ▶ Conductivity normal to the body surface.

q_S ▶ Imposed heat flux on S_q .

Convection conditions :

h ▶ Convection coefficient

θ_e ▶ External temperature

θ_S ▶ Surface temperature

Radiation conditions :

θ_r ▶ temperature of radiating source

κ ▶ radiating coefficient, variable with temperature according to the following law :

$$\varepsilon \text{ ▶ emissivity} \quad \kappa = \varepsilon \cdot \sigma \cdot f \cdot \left(T_r^2 + (T^S)^2 \right) (T_r + T^S)$$

σ ▶ Stefan-Boltzmann constant

f ▶ view factor.

$$T|_S = T^S$$

$$k_n \frac{\partial T}{\partial n} \Big|_{S_q} = q^S$$

$$q^S = h(T_e - T^S)$$

$$q^S = \kappa(T_r - T^S)$$

THERMOELASTIC EQUATIONS

Governing equations for a body of volume V a boundary S in a 3D space

$$\tau_{ij,i} + f_j^B = 0 \quad \longrightarrow \quad \text{Mechanical equilibrium}$$

$$\varepsilon_{ij} = 1/2 (u_{i,j} + u_{j,i}) \quad \longrightarrow \quad \text{Compatibility equations}$$

$$(k_{ij} T_{,rj})_{,ri} = -q^B \quad \longrightarrow \quad \text{Fourier law}$$

in V

$$n_i \tau_{ij} = f_j^S \quad \longrightarrow \quad \text{Natural mechanical conditions on } S_f$$

$$k_n dT/dn = q^S \quad \longrightarrow \quad \text{Natural thermal conditions on } S_T$$

$$u_i = \hat{u}_i \quad \longrightarrow \quad \text{Essential mechanical conditions on } S_U$$

$$T = \hat{T} \quad \longrightarrow \quad \text{Essential thermal conditions on } S_T$$

on S

$$\tau_{ij} = C_{ijhk} \varepsilon_{hk} - C_{ijhk} \alpha_{hk} \Delta T \quad \longrightarrow \quad \text{Constitutive ("mechanical")}$$

$$q_{,i} = -k_{ij} dT/dx_j \quad \longrightarrow \quad \text{equations ("thermal")}$$

VARIATIONAL PRINCIPLES (1)

VIRTUAL WORKS (displacement version)

$$L_{ext} = L_{int}$$

$$\int_V \sigma_{ij} \tilde{\epsilon}_{ij} dV = \int_V X_i \tilde{u}_i dV + \int_{S_f} f_i \tilde{u}_i dS$$

σ_{ij} : Actual stress

X_i, f_i : Actual external forces

\tilde{u}_i : Virtual displacements (small, zero on S_u)

$\tilde{\epsilon}_{ij}$: Virtual strain (Compatible with \tilde{u}_i)

WEAK FORM EQUILIBRIUM EQUATION:

$$\int_V (\sigma_{ij,j} + X_i) \tilde{u}_i dV = 0 \quad i, j=1,2,3$$

$$\sigma_{ij,j} \tilde{u}_i = (\sigma_{ij} \tilde{u}_i)_{,j} - \sigma_{ij} \tilde{u}_{i,j} \quad \text{Derivative of a product}$$

$$\int_V (\sigma_{ij} \tilde{u}_i)_{,j} dV = \int_S \sigma_{ij} \tilde{u}_i n_j dS \quad \text{Divergence theorem} \quad S=S_u+S_f$$

$$\sigma_{ij} n_j = f_i \quad \text{on } S_f$$

$$-\int_V \sigma_{ij} \tilde{u}_{i,j} dV + \int_{S_f} f_i \tilde{u}_i dS + \int_V X_i \tilde{u}_i dV = 0$$

$$\sigma_{ij} \tilde{u}_{i,j} = \sigma_{ij} \frac{1}{2} (\tilde{u}_{i,j} + \tilde{u}_{j,i}) = \sigma_{ij} \tilde{\epsilon}_{ij} \quad \text{For the symmetry of } \sigma$$

VARIATIONAL PRINCIPLES (2)

VIRTUAL WORKS EQUATIONS

$$\int_V \overset{\text{STRESS}}{\sigma_{ij}} \overset{\text{VIRTUAL STRAIN}}{\tilde{\epsilon}_{ij}} dV = \int_{S_f} \overset{\text{SURFACE FORCES}}{f_i} \overset{\text{VIRTUAL DISPL.}}{\tilde{u}_i} dS + \int_V \overset{\text{BODY FORCES}}{X_i} \tilde{u}_i dV$$

VARIABLES INVOLVED IN FIELD PROBLEMS

•STATE VAR.	•HEAT COND.	•MOISTURE COND.	•ELECTROSTATICS
•“DISPL.” u_i	•TEMPERATURE • $T(x_i, t)$	•MOISTURE CONTENT $U(x_i, t)$	•ELECTRIC POTENTIAL $\Phi(x_i, t)$
•“STRAIN.”	•TEMP. GRADIENT $T_{,j}(x_i, t)$	•MOIS. CONT. GRAD. • $U_{,j}(x_i, t)$	•ELECTRIC FIELD • $E_i(x_i, t)$
•“STRESS”	•HEAT FLUX $q_i(x_i, t)$	•HUMIDITY FLUX $q_i(x_i, t)$	•ELECTRIC DISPLACEMENT $D_i(x_i, t)$

VARIATIONAL PRINCIPLES (3)

VIRTUAL WORKS EQUATIONS

$$\int_V \sigma_{ij} \tilde{\epsilon}_{i,j} dV = \int_{S_f} f_i \tilde{u}_i dS + \int_V X_i \tilde{u}_i dV$$

↑ STRESS
↑ SURFACE FORCES
↑ BODY FORCES

↓ VIRTUAL STRAIN
↓ VIRTUAL DISPL..

EXTERNAL ACTIONS

•“BODY FORCES”	•HEAT GENERATED PER UNIT VOL. q^B	•GENERATED MOIST. PER UNIT VOLUME q^B	•ELECTRIC CHARGE PER UNIT VOL. $\sigma^B=0$
•“SURFACE FORCES”	•HEAT FLUX IMPOSED ON BOUNDARY q^S	•MOIST. FLUX IMP. ON BOUNDARY q^S	•IMPOSED ELECTRIC SURFACE CHARGE $\sigma^S=\sigma^S$
•“IMPOSED DISPL.”	•IMPOSED TEMP. ON BOUNDARY \bar{T}	•HUMIDITY CONT. IMP. ON BOUNDARY \bar{U}	•IMPOSED ELECTRIC POTENTIAL $\Phi_i = \Phi_i$

VARIATIONAL PRINCIPLES (4)

FIELD EQUATIONS	SOLID MECH.	HEAT COND..	ELETTROSTATICS
<i>EQUILIBRIUM</i>	$\sigma_{ij} + X_i = 0$	$-q_{i,j} = c\rho \frac{\partial T}{\partial t} - c$	$D_{i,i} = 0$
	$X_i = \hat{X}_i - \rho \ddot{x}_i$	$n_i q_i = q^n$	$n_i D_i = \sigma^s$
<i>COMPATIBILITY</i>	$\epsilon_{ij} = 1/2(u_{i,j} + u_{j,i})$	$\text{grad } T = T_{,j}$	$E_i = -\Phi_{,i}$
	$u_i = \hat{u}_i$	$T = \bar{T}_i$	$\Phi_i = \bar{\Phi}_i$

VIRTUAL TEMPERATURE EQUATION

$$\int_V q_i \tilde{T}_{,i} dV = \int_V q_B \tilde{T} dV + \int_{S_{qs}} q_s \tilde{T} dS$$

“STRESS”
“BODY FORCES”
“SURFACE FORCES”

“VIRTUAL STRAIN”
“VIRT. DISPL..”

VIRTUAL ELECTRIC POTENTIAL EQUATION

$$\int_V D_i E_i dV = - \int_{S_{os}} \sigma_s \tilde{\Phi} dS \quad \sigma_b = 0$$

“STRESS”
“SURFACE FORCES”

VARIATIONAL PRINCIPLES (5)

VIRTUAL WORK:

$$\int_V \sigma_{ij} \tilde{\varepsilon}_{ij} dV = \int_V X_i \tilde{u}_i dV + \int_{S_F} F_i \tilde{u}_i dS$$

VIRTUAL TEMPERATURE:

$$\int_V q_i \tilde{T}_{,i} dV = \int_V q_B \tilde{T} dV + \int_{S_{qs}} q_S \tilde{T} dS$$

CONSTITUTIVE EQUATIONS

Elastic body:

$$\sigma_{ij} = C_{ijhk} \varepsilon_{hk} \quad \text{or} \quad \varepsilon_{ij} = F_{ijhk} \sigma_{hk}$$

Thermal body:

$$q_i = -k_{ij} T_{,j}$$

Thermoelastic body:

$$\varepsilon_{ij} = F_{ijhk} \sigma_{hk} + \alpha_{ij} \Delta T$$

Virtual work equations are coupled with virtual temperature equations by means of const. Eq.

VARIATIONAL PRINCIPLES (6)

NONLINEAR CASE

For every t and δu_i respecting essential bc, the following eq. holds

$$\int_V {}^t \tau_{ij} \delta_t \varepsilon_{ij} d^t V = \int_V {}^t f_i^B \delta u_i d^t V + \int_{S_f} {}^t f_i^S \delta u_i d^t S$$

where $\delta_t \varepsilon_{ij} = 1/2(\delta u_{i,j} + \delta u_{j,i})$

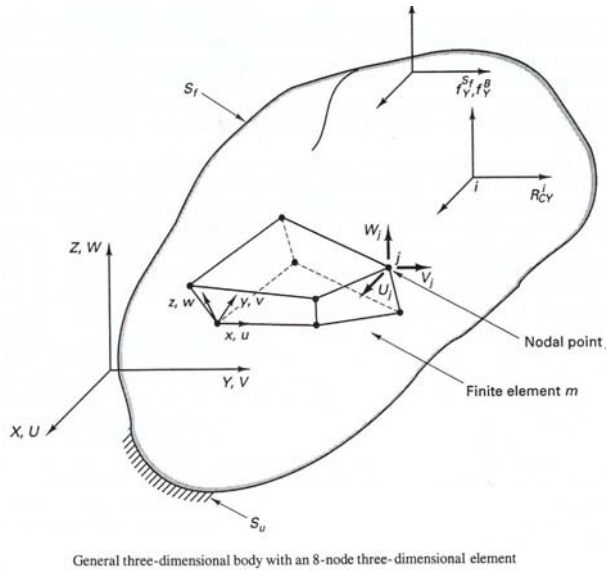
In an analogous way it can be written

$$\int_V {}^t q_i \delta T_{,i} d^t V = \int_V {}^t q_B \delta T d^t V + \int_{S_{qs}} {}^t q_S \delta T d^t S$$

where $\delta T_{,i}$ is the temperature gradient corresponding to δT .

No restriction is introduced by the governing variational principles, giving the procedure a high degree of generality.

Discretization



A fundamental ingredient of the finite element method is the discretization process. The original volume of the body is divided into elemental parts over which it is easy to formulate an hypothesis of interpolation for the variables of the problem. The contribution coming from each element are then combined for generating the global finite element equations. In fact the virtual work equation (or their equivalent) is written for every element and then combined for obtaining the resulting general solving equations.

LINEARIZATION AND FINITE ELEMENT INTERPOLATION (1)

MECHANICAL PART

An incremental procedure is set up:

1) at $t+\Delta t$:

$${}^{t+\Delta t} R_e - {}^{t+\Delta t} R_i = 0$$

Where R_e are the external forces supposed to be known for every t and R_i are the internal forces; for them it is assumed that:

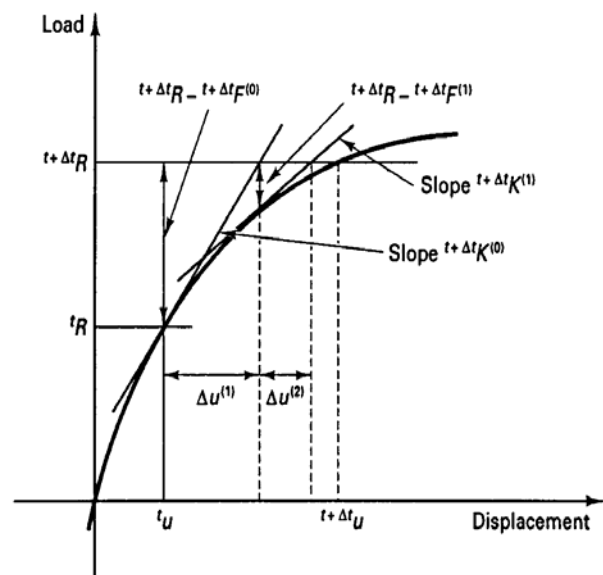
$${}^{t+\Delta t} R_i - {}^t R_i \cong {}^t K \Delta u_i \quad \Delta u_i = {}^{t+\Delta t} u_i - {}^t u_i$$

Then for successive approximations the following equations are solved

$${}^t K \Delta u_i = {}^{t+\Delta t} R_i - {}^t R_i$$

Typical interpolation functions are assumed inside every element m :

$$\underline{u}^{(m)} = \underline{H}_u^{(m)} \underline{u}$$



LINEARIZATION AND FINITE ELEMENT INTERPOLATION (2)

THERMAL PART

Similar assumptions are done for the thermal part

$${}^{(i)}\Delta T = {}^{t+\Delta t} T^{(i-1)} - {}^{t+\Delta t} T^{(i)}$$

where also the iteration inside the single step ΔT has been introduced. ${}^{t+\Delta t} T^{(i-1)}$ is the temperature obtained at the end of $(i-1)$ th iteration ${}^{(i)}\Delta T$ is the temperature increment for iteration (i) . Moreover

$${}^{t+\Delta t} T^{(0)} = {}^t T$$

In both cases such approximations are introduced in the virtual work equations obtaining in this way a system of linear algebraic equations For each step or, for linear response, for the overall loading condition.

FINITE ELEMENT EQUATIONS

The linearization allow us to write the equation in the following form:

$$\int_V \tilde{\underline{T}}^T \underline{\underline{K}}^{(i-1)} \underline{\Delta T}^{(i)} dV + \int_{S_c} \tilde{\underline{T}}^S \underline{h}^{(i-1)} \Delta T^{S(i)} dS + \int_{S_r} \tilde{\underline{T}}^S \tilde{\underline{\kappa}}^{(i-1)} \Delta T^{S(i)} dS =$$

$$L + \int_{S_c} \tilde{\underline{T}}^S \underline{h}^{(i-1)} (T_e - T^{S(i-1)}) dS + \int_{S_c} \tilde{\underline{T}}^S \underline{\kappa}^{(i-1)} (T_e - T^{S(i-1)}) dS - \int_V \tilde{\underline{T}}^T \underline{\underline{K}}^{(i-1)} \underline{T}^{(i-1)} dV$$

where: $\tilde{\underline{\kappa}} = 4 \cdot \underline{\varepsilon} \cdot \underline{\sigma} \cdot \underline{f} \cdot (T^S)^3$

Assume that the body has been discretized into m elemens. The temperature in a specific point is obtained by interpolating the nodal variables :

$$\underline{T}^{(m)} = \underline{\underline{H}}^{(m)} \underline{T}$$

$$\underline{T}^{S(m)} = \underline{\underline{H}}^{S(m)} \underline{T}$$

$$\underline{T}^{\prime(m)} = \underline{\underline{B}}^{(m)} \underline{T}$$

\underline{T} ▶ nodal temperature vector

$\underline{H}^{(m)}$ ▶ temperature interpolation (shape functions) matrix

$\underline{B}^{(m)}$ ▶ temperature gradient interpolation (shape functions) matrix

Finite elements equations (2)

Linear or linearized case

$$\left(\underline{\underline{K}}^K (i-1) + \underline{\underline{K}}^c (i-1) + \underline{\underline{K}}^r (i-1) \right) \underline{\underline{\Delta T}}^{(i)} = \underline{\underline{Q}} + \underline{\underline{Q}}^c (i-1) + \underline{\underline{Q}}^r (i-1) - \underline{\underline{Q}}^K (i-1) \quad (\text{A})$$

CASCADE COUPLING

$$\underline{\underline{K}}_u \underline{\underline{\Delta u}} = \underline{\underline{F}}^S + \underline{\underline{F}}^B + \underline{\underline{F}}^T$$

$\underline{\underline{\Delta u}}$ and $\underline{\underline{\Delta T}}$ are, respectively, the increment in nodal displacements and nodal temperatures, $\underline{\underline{Q}}$ are heat fluxes, $\underline{\underline{F}}^S$, $\underline{\underline{F}}^B$ are the surface and body forces $\underline{\underline{F}}^T$ the induced thermal forces .

$$\begin{aligned} \underline{\underline{K}}_u &= \sum_m \int_{V(m)} \underline{\underline{B}}_u^{(m)T} \underline{\underline{C}} \underline{\underline{B}}^{(m)} dV \\ \underline{\underline{F}}^S &= \sum_m \int_{S(m)} \underline{\underline{H}}^{S(m)T} \underline{\underline{f}}^{S(m)} dS \\ \underline{\underline{F}}^B &= \sum_m \int_{V(m)} \underline{\underline{H}}^{(m)T} \underline{\underline{f}}^{B(m)} dV \end{aligned}$$

The summation is extended to all the elements of the discretization.. The shape functions are embedded in the matrices $\underline{\underline{H}}$; the matrices $\underline{\underline{B}}$ are obtained from $\underline{\underline{H}}$ by derivation.

Finite elements equations (3)

Linear or linearized case

$$\begin{aligned} \underline{\underline{K}}^{k(i-1)} &= \sum_m \int_{V(m)} \underline{\underline{B}}^{(m)T} \underline{\underline{k}}^{(m)(i-1)} \underline{\underline{B}}^{(m)} dV \\ \underline{\underline{K}}^{c(i-1)} &= \sum_m \int_{S_c(m)} h^{(m)(i-1)} \underline{\underline{H}}^{S(m)T} \underline{\underline{H}}^{S(m)} dS \\ \underline{\underline{K}}^{r(i-1)} &= \sum_m \int_{S_r(m)} \tilde{\kappa}^{(m)(i-1)} \underline{\underline{H}}^{S(m)T} \underline{\underline{H}}^{S(m)} dS \\ \underline{\underline{Q}}^{c(i-1)} &= \sum_m \int_{S_c(m)} h^{(m)(i-1)} \underline{\underline{H}}^{S(m)T} \left[\underline{\underline{H}}^{S(m)} (\underline{\underline{T}}_e - \underline{\underline{T}}^{(i-1)}) \right] dS \\ \underline{\underline{Q}}^{r(i-1)} &= \sum_m \int_{S_r(m)} \kappa^{(m)(i-1)} \underline{\underline{H}}^{S(m)T} \left[\underline{\underline{H}}^{S(m)} (\underline{\underline{T}}_r - \underline{\underline{T}}^{(i-1)}) \right] dS \\ \underline{\underline{Q}}^{k(i-1)} &= \sum_m \int_{V(m)} \underline{\underline{B}}^{(m)T} \left[\underline{\underline{k}}^{(m)(i-1)} \underline{\underline{B}}^{(m)} \underline{\underline{T}}^{(i-1)} \right] dV \\ \underline{\underline{Q}} &= \sum_m \int_{S(m)} q^{S(m)} \underline{\underline{H}}^{S(m)T} dS + \sum_m \int_{V_B(m)} q^{B(m)} \underline{\underline{H}}^{(m)T} dS \end{aligned}$$

Analytical solutions and numerical simulation Special topics

- Solution techniques: displacements method or force method
- Semi-inverse method
- Existence of solution
- Linear and nonlinear problems
- Convergence
- Coupled behaviour (and behaviour of interfaces)
- The modeling of space environment

Models for structures

- Structural models (bar, cable, beam, plate, shell, membrane)

CONCLUDING REMARKS

Modelling statics, dynamics, thermoelastic responses

Continuous vs lumped, linear vs nonlinear.

Mathematical bases of the finite element methods.

Elements of nonlinear analysis