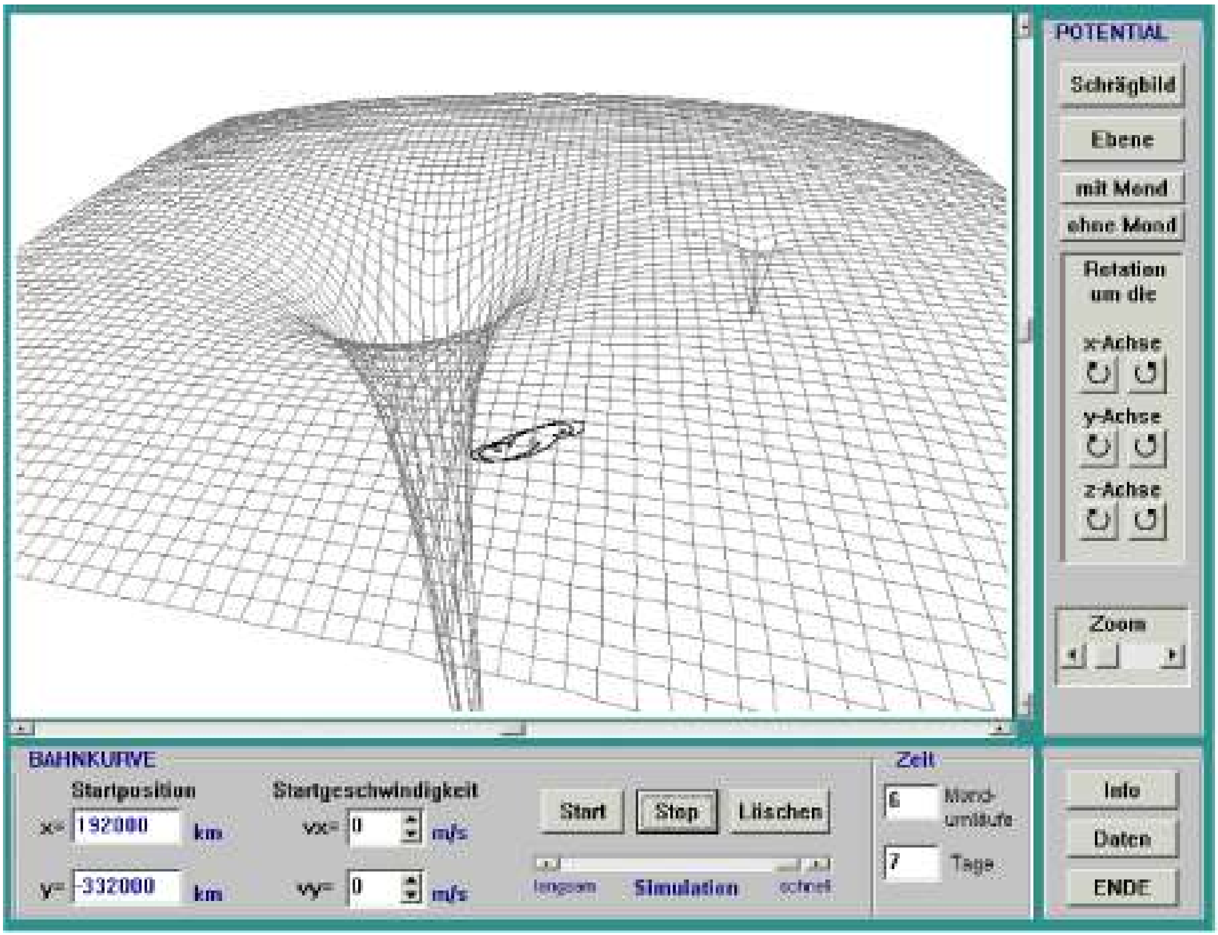
## The simulation program ‘lagrange.exe’

Description:

The program calculates the two gravitational forces and the centrifugal force exerted at the location of a space probe and thus determines the direction and amount of acceleration and speed of the probe and hence its new position after a period of time $\Delta t$ (Euler-Cauchy method).
However only those paths that lie on the plane of the Earth, the centre of gravity and the Moon are calculated. It is therefore not possible to follow the path of a probe that has been placed above or below this plane.
From a teaching point of view this reduction is especially advantageous as the relationships between the forces are still conceivable to some degree. There is an additional advantage due to the possibility of mapping the path of the space probe onto the surface of the gravitational potential (‘potential surface’) and to draw the entire thing as an axonometric projection. This creates a very vivid impression of the topology of the energetic relationships in the rotating Earth-Moon system.
The coordinate system in which the space probe’s movements are calculated and drawn rotates around the common centre of gravity together with the Earth-Moon system. This means that the Earth and the Moon are resting in this reference system, which allows a clear representation of the space probe’s path as we can see how it moves only in relation to the Earth and the Moon.

Operation:



1. The **potential energy surface** is shown as the axonometric projection of a grid. This diagram can be changed (turned and zoomed in on) using the operating controls in the right-hand ‘POTENTIAL’ bar. In addition we can demonstrate the energy potential relationships without the Moon, i.e. without centrifugal potential, as this is only created due to the rotation of the Earth and the Moon around the common centre of gravity.
Here we can clearly see how the potential energy surface warps outwards to become convex as soon as we start the rotation (i.e. of the Moon).

2. The **path** of the space probe is pictured on the potential energy surface. Start conditions, i.e. the coordinates of the start position and the start speed relative to the coordinates x and y, can be set (lower ‘BAHNKURVE’ [PATH] bar). One start position is already preset when the program is opened - a position in the vicinity of the Lagrange point L5. These coordinates can of course be overwritten. To obtain an overview of possible start values you can see the coordinates of the 5 Lagrange points under *‘Daten’ [Data]*.

Examples:

- Change the pre-set coordinates of the starting point to be in the vicinity of L5 or L4. The looped curves become smaller in size or are further elongated, but retain a similar shape. Only when the paths protrude so much that they fall into the potential vortex of the Earth or the Moon are chaotic path shapes created.
- Investigate the behaviour of the point L1 by entering a start value that is somewhat to the left of the Lagrange point (e.g. x=321688 km, y=0 km) and then somewhat to the right. (e.g. x=321689 km, y=0 km). We can easily see from the paths that L1 does not allow a stable equilibrium - the space probe drifts to the left or to the right and the Coriolis force causes it to meander instead of following a linear path.
- This kind of unstable behaviour can also be observed at points L2 and L3.
- Select as start parameters: x=240000 km, y =0, vx=0, vy=0. The space probe does not fall straight into the potential vortex of the Earth - rather, the Coriolis force causes curves that direct it out of the vortex again.
- Select *‘ohne Mond’ [without Moon]* and start with: x=240000 km, y =0, vx=0, vy=700 m/s. The space probe moves on an elliptical path as can be expected for a two-body problem. The elliptical shape is easily identifiable if you click on *‘Ebene’ [Level]* as you can then see the situation from above.

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