The Lagrange points - more than simply solutions to a differential equation

Experiment: Lissajous figures

Materials for the experiment:
We need a cup with a hole or a funnel, some easily pourable salt, thread, a freezer clip, a rod (mounted horizontally, on which the pendulum will be hung) or two hooks secured to the ceiling at a distance of approximately 50cm, a black blanket or a sheet of black drawing paper as an underlay, a stopwatch, a cm gauge and a camera.

Experiment 1

Design: We are going to make a pendulum on a thread that can swing freely above a black blanket or drawing paper and which is secured to a hook in the ceiling. As a weight we will use a paper cup filled with pourable salt or sand which has a small hole in the bottom (must not be too small).

Method: We will allow the pendulum to swing both one-dimensionally and two-dimensionally.

Evaluation: The oscillation amplitudes (X_o, Y_o) can be measured from the dimensions of the path of salt in the direction of both x and y, and we will use a stopwatch to measure the period of oscillation T . This provides the oscillation equations

$$x(t)=X_0\cdot\sin(\omega t+\phi_1)$$
$$y(t)=Y_0\cdot\sin(\omega t+\phi_2)$$
$$\text{where}\quad \omega=\frac{2\pi}{T}\quad .$$

The period of oscillation T for a pendulum on a thread with length l can be calculated using

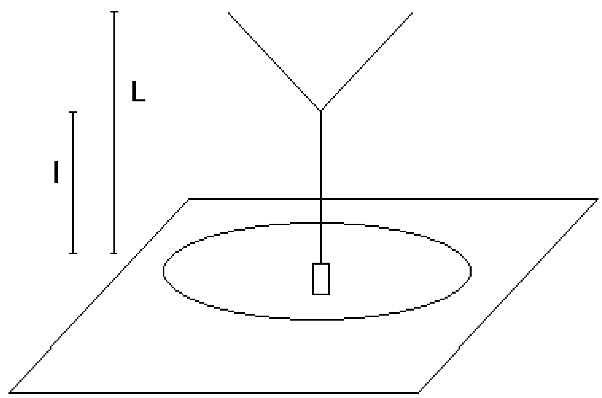
$$T=2\pi\sqrt{\frac{l}{g}}\quad .$$

The pendulum swings with the same frequency in both directions, as this depends only on the length of the thread!

Experiment 2

Now we want the oscillations in the direction of x and y to have different frequencies. To do this we have to be able to vary the pendulum lengths for each direction independently of one another. This is done using a linked pendulum.

Design: We will attach one thread to each of the two hooks in the ceiling or to the ends of the suspended rod. Then we will clip together the two threads that are dangling down such that they form a triangle above the clip. We will again attach our salt-filled cup to the lower ends. Now the system can swing in the one direction with thread length L (ceiling to cup) and the frequency $f(L)=1/T(L)$, but at right angles to this it can only swing with the thread length l (clip to cup) and the associated frequency $f(l)=1/T(l)$, because the thread triangle prevents the swinging from happening as per the above test set-up.



Method: Make different salt paths by varying the lengths of the thread using the freezer clip.

Evaluation: Document your results using a camera. Compare your pictures with those in the references.