The physics of background radiation - ideas for years 10-12

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Cosmology - a topic top motivate your students in physics class! Anyone who has previously discussed the development and creation of the Universe in a lesson is familiar with the innumerable questions from students generally associated with this material. But they would also be familiar with the disheartening feeling of not bringing enough physics into the discussion with the students, and of simply reproducing the same level of knowledge that has been published in innumerable popular science articles. You can use final year-level physics to discuss a whole range of cosmological issues based on physical laws and even solve problems. Below we provide several examples which chiefly relate to cosmological background radiation.

Overview of references		
Physics	Thermodynamics, theory of relativity, quantum physics	Collision of particles, first law, adiabatic changes of condition, thermal radiation, black bodies, Planck's radiation law, Stefan-Boltzmann Law, light propagation, Bohr atomic model, ionization, Compton effect and other scattering effects, WW between electrons and radiation
Astronomy	Cosmos	Creation of background radiation, expansion of the Universe, cooling of background radiation, structures in background radiation
Related disciplines	Mathematics	Functions

Adiabatic changes of condition - analogy of an air pump and the Universe

Thermodynamic systems have system boundaries. Outside these system boundaries the system can exchange either heat Q or mechanical work W with its surrounding environment and as a result can change its internal energy ΔU . The change in system energy is expressed by an increase or decrease in temperature T. This correlation, which is called the first law of thermodynamics, can be written as an equation:

$$\Delta U = Q + W \qquad (1)$$

Wherever thermodynamic processes take place very quickly in our everyday environment there is some probability that these are so-called adiabatic changes of condition. During such adiabatic changes of condition the system does not exchange any warmth with the surrounding environment - simply because it 'does not have time' to do so. The gas in an air pump that is operated very quickly with the valve pressed closed becomes hot. It is (almost) undergoing an adiabatic change of condition. If $Q=0$ *for the heat,* then the first law of thermodynamics states that

$$\Delta U = W \qquad (2)$$

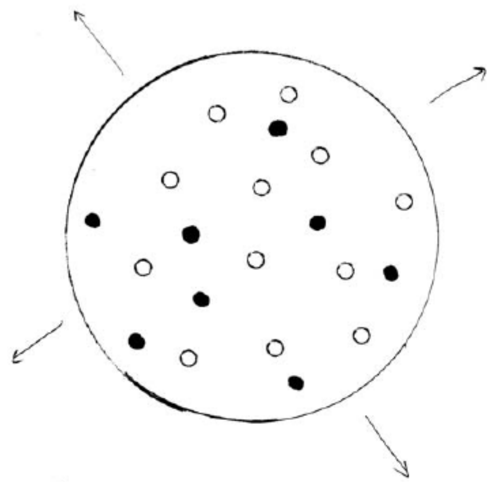
Expressed in words: if you do mechanical work with a system, its level of internal energy increases - the system becomes hotter. Viewed from a different angle: if the system does mechanical work to its environment, its level of internal energy decreases, i.e. the system becomes colder. These insights deal mainly with final year-level physics material.

Exercise 1
State and explain some examples of adiabatic changes of condition in the areas of nature and technology.

Solution
Did the examples that you mentioned in exercise 1 also include the Universe? Our Universe is expanding. In the process it obviously needs to do work in order to overcome the gravitational attraction of the matter. But does heat flow away across any system boundaries? It would appear that within *one* Universe it is only possible for heat to be exchanged between different areas *within* the system. However such a process would contradict a central claim that has until now been made in the area of cosmology with great success and which is referred to as a cosmological principle: no part of the Universe looks any different from any other part in any way. Therefore, the Universe must look the same throughout (spread over large distances). We could measure it in any direction and we would determine the same characteristics as in any other direction. However if in the past heat had flowed in a certain direction from one part of the Universe to another, it would have had a certain special direction such as this. It is a contradiction to the assumption we made at the outset, such that we have good reason to assume that our Universe is undergoing an adiabatic change of condition as it expands.

If our Universe were to contract it would inevitably have to heat up - like the gas in the air pump whose piston is pushed in quickly. On the other hand, during an expansion it must cool down - but only when work is also done as it diverges (see first law). The analogy between the Universe and the air pump is not a formal one - from a thermodynamic point of view both systems actually do display a physical behaviour that can be described by similar basic thoughts.

Densities of energy and mass in the Universe
Let's consider a small section of our Universe that, due to the cosmological principle, must have exactly the same characteristics as any other part of the Universe and let's give this small section the shape of a sphere. In this way we can create - to no greater or lesser extent - an analogy with the gas model container that is well-known from school level physics. For an adiabatic change of condition to be able to take place in the container we must insulate it well from its surrounding environment, preferably using a mirror lining such as can be found in thermos bottles. In addition its size must be able to be changed. Now we will fill the sphere with the two main components of our Universe - hydrogen atoms and electromagnetic radiation (photons) - and allow the sphere to expand (Fig. 1). For our purpose it is helpful to consider the radiation and the atoms separately.



o hydrogen atoms
• Photons

Figure 1: Photons • and hydrogen atoms o diverge and cause a container to expand. In the process they must do expansion work.

The matter density ρ_M of the hydrogen atoms is simply the quotient of the mass m of all of the particles in the sphere-shaped container and the volume *V*, i.e.

$$\rho_M = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} \sim \frac{1}{R^3}, \quad (3)$$

where *R* is the radius of the sphere. In order to investigate the radiation density we will consider very special sections of radiation. We will first concentrate on stationary waves, i.e. those electromagnetic sections of radiation for which exactly a fixed multiple of wave crests and wave troughs fits between the vessel walls such that they are reflected back onto the mirrored walls.

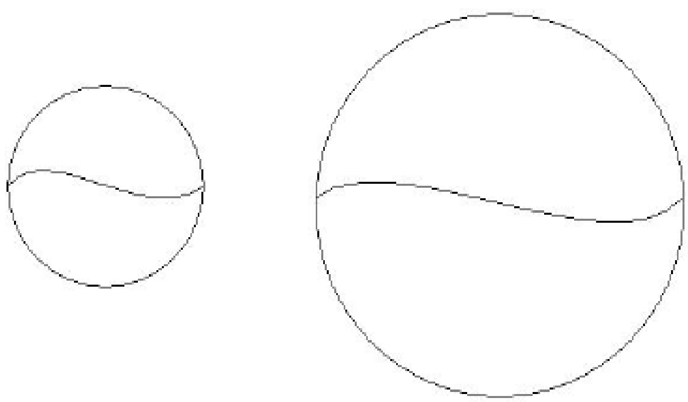


Figure 2: Expansion model for stationary waves in the container.

If the distance between the walls is now gradually expanded, the wavelength of these stationary waves will slowly increase (Fig. 2). This effect will also occur for those wave trains in the electromagnetic radiation that have not randomly formed a stationary wave in the vessel. However, using the stationary wave trains it is easy to tell what is happening to the energy in the electromagnetic radiation. The wavelength increases in proportion to the diameter (and radius) of our sphere-shaped vessel:

$$\Lambda \sim R\,. \qquad (4)$$

According to Einstein's light quantum hypothesis we can imagine that the radiation consists of photons - the radiation quantums - each of which possesses a certain energy *E* of quantity *E=hf (h:* Planck's quantum of action, / frequency). This frequency is connected to the wavelength of the photons via the basic relationship *c=λf (c:* speed of light). Thus we obtain the proportionality chain for the correlation between the energy of the photons and the radius of our vessel:

$$E = hf = \frac{hc}{\lambda} \sim \frac{1}{\lambda} \sim \frac{1}{R}. \qquad (5)$$

On the other hand the energy density of the radiation *u* is of course proportional to *1/R3* , such that if we take into account the correlation (5) we obtain the following for the overall radiation energy density:

$$u \sim \frac{1}{R^4}\,. \qquad (6)$$

In order to convert an energy density *u* to a radiation mass density *ρS* we can use the famous correlation $E=mc^2$ and we obtain

$$\rho_S = \frac{m}{V} = \frac{E}{c^2 V} = \frac{u}{c^2} \sim u \sim \frac{1}{R^4}\,. \qquad (7)$$

Because we had mirrored the inner walls of our small sphere, no heat was able to escape from the partial volumes. We have thus in actual fact observed an adiabatic change of condition and can assume that the matter and the radiation behave in the same way in space as in the relatively small partial volumes that we have just analysed - even without the vessel walls. *R* would then be the radius of a certain area within the Universe, recognisable e.g. as a distance between two distant galaxies.

If *R increases,* the matter density decreases by *1/R3* , but the radiation (mass) density decreases by $1/R^4$. So the radiation density decreases much more quickly than the matter density.

The current known density of the visible matter in the Universe is around ~10^{-27} kgm^{-3}, which corresponds to roughly one hydrogen atom per cubic metre. The density of the background radiation can be calculated from its temperature (see further below) of around 3 K. It is ~10^{-30} kgm^{-3}.

Exercise 2
Calculate by what factor R would have to decrease for matter and radiation to have the same density in our Universe.

Solution
From the relationships (3) and (7) we obtain the equation

$$\alpha = \frac{\rho_S}{\rho_M} = \frac{R^3}{R^4} = \frac{1}{R}. \qquad (8)$$

At present the relationship α between radiation density and matter density is 10^{-3}. If we denote this density relationship using α_G, a density relationship that was present in the Universe in the past using α_V and use R_G and R_V for the associated radii of the partial volumes that we are observing, from equation (8) we obtain the correlation

$$\frac{\alpha_V}{\alpha_G} = \frac{R_G}{R_V} \quad \text{or converted} \quad \alpha_V = \alpha_G \frac{R_G}{R_V}. \qquad (9)$$

If $\alpha_V = 1$, the numerical value of 1000 is obtained from $\alpha_G = 10^{-3}$ for the quotients R_G/R_V . At the time when matter density and radiation density were the same the radius *R* (and therefore the entire size of the Universe) was 1000 times smaller than today's figure.

The recombination and the ionisation of hydrogen atoms in the Universe
What happens to matter when it is thrown into a hot radiation bath which has a very high density? The matter dissolves into its elemental components, similar to a lump of sugar in coffee. First the connections between the atoms are dissolved - all of the molecules disassociate. If the density of the radiation increases further, the atoms are ionised. In the process the electrons absorb the energy from individual photons which must be sufficiently large to overcome the electrical pull of the atomic nucleus. The photons are absorbed by the electrons. If the radiation density is sufficiently high (‚radius' of the Universe is sufficiently small) this photon energy that we saw in equation (6) and (7) is correspondingly high.

Exercise 3

Calculate the minimum amount of energy a photon must have in order to ionise a hydrogen atom.

Solution

It is impossible for us to repeat the entire syllabus of final year-level nuclear physics here, but we can say this much: to calculate the ionisation energy of a hydrogen atom we must remember the Bohr atomic model: the shell electron is raised from a lower path to a higher path when it absorbs a photon that has exactly the same amount of energy as the difference in energy between the two paths. Only paths with an integral path index are allowed. Using the so-called Rydberg frequency (R_H=3.29-10^{15} Hz) the following holds for photon energy when it changes from the nth to the mth path:

$$E = hf = hR_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right) . \qquad (10)$$

If we consider the ionisation of a hydrogen atom from its basic state, then *n=1* and m=∞ and from (10) we derive

$$E = hR_H = 2,2^{-18}\,J = 13,6eV . \qquad (11)$$

The background radiation and the Planck distribution

Thermodynamics provides us with an answer to the question of what we can expect to happen to the temperature of particles in a gas. We can view the temperature as an expression of the movement of the particles. In a gas with a high temperature the particles move quickly, and if the temperature falls the particles slow down. However, not all particles have the same speed - as if on command. In addition, there are in fact too many particles that move at a speed that is of a similar magnitude to the average speed, and there are also always a few particles that move at very high speeds. If we consider a gas in a closed vessel, we can determine the temperature of the gas if we know how many particles in this gas are moving about at what speeds. For known atomic or molecular masses, knowing the particle speed is equivalent to knowing the kinetic energy of the gas particles.

Instead of the atomic gas ‹trap’ some photons from the surrounding environment and shut them inside the vessel. Measuring the speeds of the photons clearly does not help us to ascertain anything about their temperature. As we know, all photons move at the same speed - the speed of light. For this reason we can only determine how all of the energy present in the container is distributed among the photons. And because the energy of the photons is clearly determined by its frequency due to the equation E=hf, we can just as well ask how the frequency is distributed among the individual photons.

However even if we succeeded in determining the frequencies of the individual photons in the vessel we still would not be able to come to a conclusion about their temperature. After all, when we trapped the photons inside the vessel we caught a whole conglomeration - some photons from the heater under the window, some from the desk lamp, still others from sunlight that was shining through the window, etc. All of these photons from different sources initially have nothing to do with one another and therefore quite possibly also have quite different ‹temperatures’. To circumvent this problem it is best to create the photons ourself in such a way that they are all sure to have the same temperature.

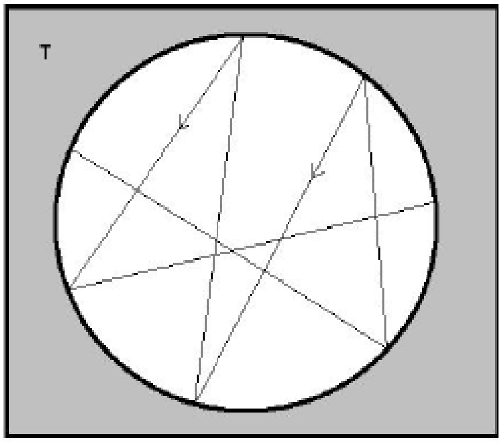


Figure 3: Model of a cavity radiator

This thought leads to the construction of the so-called cavity radiator. This is a closed vessel (Fig. 3) whose walls can be heated to a certain temperature T . Increasing the temperature of the vessel walls causes the following to occur in the internal cavity: the vessel walls emit photons into the internal cavity which is then filled with electromagnetic radiation. Because the emitted photons are just as likely to be reabsorbed by the surface, at some stage a state of equilibrium is reached. Exactly the same number of photons travel out of the vessel walls into the hollow cavity per unit of time as from the hollow cavity into the vessel walls. And because the temperature of the vessel wall no longer changes due to the absorption and emission processes we must assume that the temperature of the radiation in the hollow cavity is just as high as the temperature of the vessel walls. In the following we will exclusively address electromagnetic radiation of this type in a cavity radiator, which is also called thermal radiation. If we investigate how the photon energies (and the energy densities) of this radiation are distributed across the frequency (and wavelength) a characteristic function is obtained that is called a Planck function or Planck distribution.

Electromagnetic thermal radiation that is in thermodynamic equilibrium with its surrounding environment is very rarely found in our everyday environment as usually the equilibrium is disrupted between the emission and absorption processes. There simply are not so many natural processes that roughly resemble a cavity radiator that perfectly traps radiation. However there are many places in the Universe where this is the case - the interior of stars! The matter in the stars is mostly completely ionised and has a very high density. The electrons from the star matter, which move in large quantities, allow the radiation to pass only very very slowly from the inside of the star to the surface...
However there is a serious hitch: the Planck distribution in the interior of a star can of course hardly be measured.

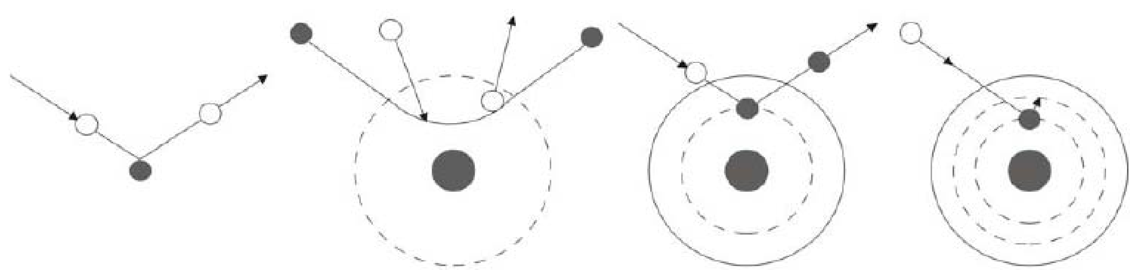
Exercise 4

From your lessons you will be aware of the various processes in which matter interacts with electromagnetic radiation (photons). Explain these processes and in particular explain their effect on the radiation.

Solution

1. *The Compton effect:* Free electrons and photons come together and bump into each other in a way that is quite similar to billiard balls. If there are enough free electrons the result is many Compton scatterings in a very small space. The photons are scattered all around but do not leave the cavity.
2. *The electrons absorb photons inside the atomic shell* (see exercise 3): the photon is ‹swallowed' inside the atomic shell by the electron - in fact, usually to be emitted again shortly thereafter. In the process the electron is lifted to a higher ‹path' (Bohr atomic model) and removed from the atom (ionisation). These processes are very effective at ensuring radiation remains ‹trapped'. On the one hand a photon is held back simply because it virtually vanishes into thin air for a certain time between being absorbed and emitted. On the other hand the re-emitted photons can fly out of the atom in any direction such that here too there is no preferred direction for the radiation.
3. *Free electrons absorb photons if they are located in the vicinity of an ion:* this process in not generally discussed in school level physics. For an electron to be able to absorb a photon it does not necessarily have to be circulating the atomic nucleus on a particular path (a pre-defined energy state). It is enough for it to be able to sense the electrical field of the atomic nucleus. This process is very effective too. It is however limited by the electron density and the temperature. For a certain electron that is moving in a hot gas to thus be able to absorb a photon it must be located in the vicinity of an atomic nucleus. Only when it comes across the next atomic nucleus in its path can it again absorb a photon. And the amount of time it takes to reach the next atomic nucleus similarly depends on the density and the speed of the electrons (their temperature). This type of photon absorption is also very effective at trapping radiation.

Fig. 4 illustrates the processes described above.



Scattering of free electrons | free-free | bound-free | bound-bound

Figure 4: Illustration of interactions between photons o and electrons •.

Let's recall our previous thoughts about matter density in the Universe. If we assume that once upon a time the Universe was in a state where the matter was completely ionised by a radiation bath, then the processes 1. and 3. would had to have occurred. In an expanding Universe the last time that this occurred was around the time when the radiation density and the matter density were the same. As the radiation density fell below the matter density there were no longer enough energy-rich photons present to continue separating the electrons from the ions. The atomic nuclei continued to catch the electrons, and the electromagnetic black body radiation that had been perfectly trapped up to that point was set free.
The entire Universe was filled with photons that had a perfect Planck distribution. With the further expansion of the Universe the wavelength of these photons changed in accordance with the correlation (4). Because this happened in the same way for all photons, the Planck distribution of the radiation remained. As we have already known for several decades, we can in fact observe this radiation today. It fills the Universe completely evenly. It is called cosmological background radiation. And the Planck satellite's mission is to investigate this exact same background radiation.

Temperature of background radiation and thermal development of the Universe

In order to measure the temperature of the background radiation we must measure its Planck distribution - i.e. we must determine the intensity of the radiation at as many different wavelengths as possible and use these measuring points to determine a Planck curve. Because every Planck curve belongs to an exactly defined temperature (Fig. 5), we can read the temperature from the curve progression. In this sense there exists a background radiation *temperature*. From previous observations we know that the temperature T of the background radiation is approximately 3K.

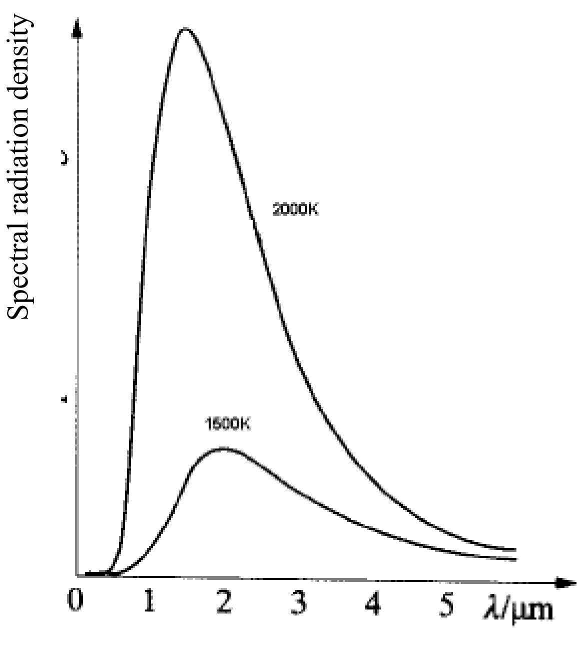


Figure 5: Examples of Planck curves.

Several years before Max Planck discovered the radiation distribution that is today named after him, the physicists Stefan (by practical means) and Bolzmann (by theoretical means) had already found out how to determine the *total* energy density of thermal radiation. The radiation energy density *u* depends only on the radiation temperature. It is proportional to the temperature to the power of four. This is the famous Stefan-Boltzmann Law

$$u \sim T^4 . \qquad (12)$$

From the Stefan-Boltzmann Law we directly obtain $\rho S \sim T^4$ for the mass density of the radiation (see (7)).

Exercise 5
Use the relationships (12) and (7) to determine how the temperature of the background radiation changes as the Universe expands.

Solution
For electromagnetic radiation R the correlation $\rho_S \sim R^{-4}$ applies. Thus if $\rho_S \sim T^4$ we directly obtain

$$T^4 \sim \frac{1}{R^4}$$

or (13)

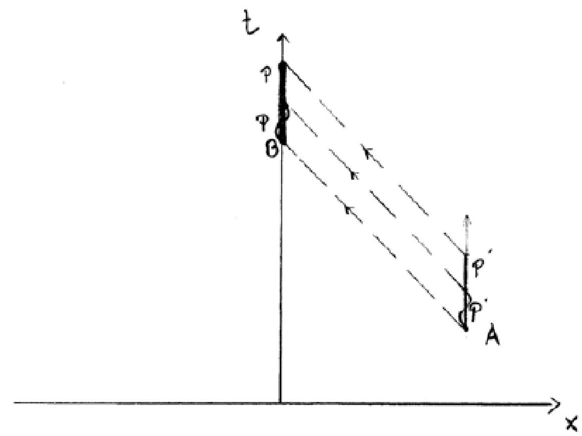
$$T \sim \frac{1}{R}$$

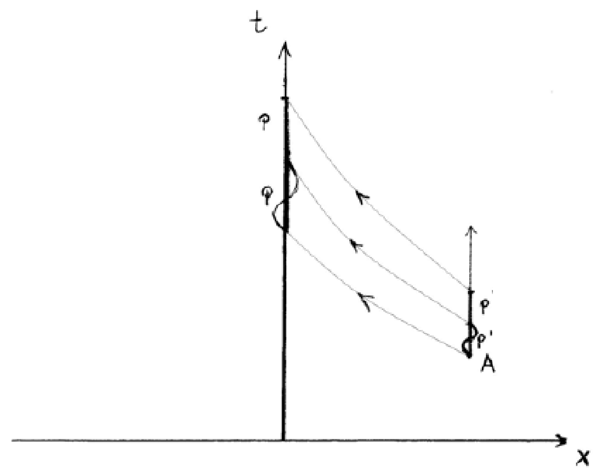
Therefore: when the matter density and the radiation density were the same in the Universe the Universe was around 1000 times smaller than it is today, and as a result the radiation temperature was around 1000 times greater than it is today. Because the radiation temperature is presently around 3K, at that time the radiation had a temperature of 3000K.

The uniformity of the background radiation
In order to become aware of an especially noteworthy characteristic of the background radiation, we must bring to mind the following facts: on the one hand, as terrestrial observers we are embedded in the background radiation and we are taking part in the cosmic expansion flow together with it; on the other hand the background radiation is made up of photons that move through space at the speed of light on paths which - at least in today's Universe - are approximately linear. The photons, which simultaneously move toward a terrestrial observer from opposing directions, therefore have their own very specific history. They do not have anything to do with each other on a causal level. Nevertheless, the photons that come from a certain direction, and which are moving in every direction, connect with the same Planck distribution.

In order to analyse this situation more closely we must expand our consideration of models and analogies that up to this point in the article were only based on thermodynamic and micro-physical considerations using some insights from the theory of relativity.

Insofar as a beam of light is not influenced by strong gravitational fields it is dispersed in a linear manner and at constant speed (of light). If we describe this dispersal in a place-time diagram we obtain a straight line (Fig. 6a). Let's assume that an intermittent light signal is emitted at a period *P* from a very distant celestial body located at *A* . This light signal first reaches the terrestrial observer at the point in time *B*. If the light signal is not bent by any gravitational fields the distance between two light pulses remains constant when it arrives at B.





Figures 6 and 6b: Light propagation in a matter-free and in a matter-filled cosmos.

Now we will fill the Universe with matter that has a higher density such that beams of light are bent in the place-time diagram. In the early Universe the density was especially high, so it is during this time that we expect this effect. Fig. 6b shows the analogous situation for this case.
The intermittent light signal is emitted at A and reaches the terrestrial observer at B. However the length of the period of time has changed! It has become greater. The period *P* can e.g. be the time that passes until two consecutive wave trains of a photon are emitted. The frequency of this photon would then have become smaller on its way to Earth and its wavelength would have become correspondingly larger. Insofar as this photon was then set free as the Universe became transparent, it is a part of the background radiation.

Exercise 6

You can use examples and your school knowledge of mathematical functions to help you understand the wavelength expansion of background radiation. To do this we have selected two quadratic functions. Draw the functions $t=(x+1)^2$ and $t=(x+1)^2+2$ in a coordinate system and determine their distance in the direction of y at the positions x=1 and x=0.

Solution

At the position x=1 the distance between the functions is $\Delta t=2$, at the position x=0 it is also $\Delta t=2$! Both functions have graphs that run parallel to the y axis. The distance has remained the same. We can learn from this insight that the bending of the various beams of light on the path from *A* to *B* must be different on order to obtain an increase in wavelength at *B* . This stands to reason: at an earlier point in time the density of the Universe was greater and the beams of light were bent differently.

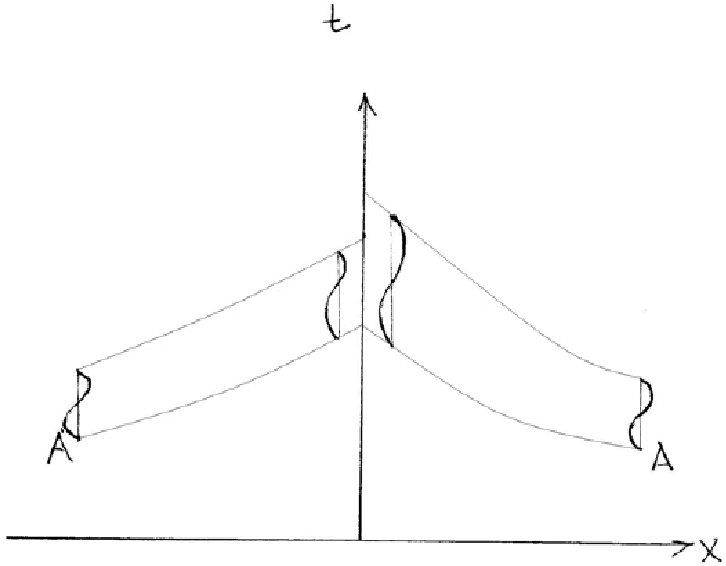


Figure 7: Beams of light in a Universe with very inhomogeneous distribution of mass

Let's now consider the background radiation in two opposing directions (Fig. 7). They are coming from *A* and from *A '*. Assume that the mass density of the early Universe at *A* ' was quite different to that in the vicinity of *A*. The beams of light would have been bent differently and the photons of the background radiation would have experienced a different type of wavelength expansion. As terrestrial observers we would construe from this that the Planck distribution of these photons had a different temperature.

Therefore, the consistent temperature of the background radiation in every direction shows that the Universe has not undergone excessively large density fluctuations when radiation and matter were separating. However if we look a little more closely at the radiation in different directions we can recognise tiny fluctuations in temperature. Therefore the density of the Universe was able to fluctuate somewhat. And it is exactly these density fluctuations that we want the Planck mission to more closely trace - after all, they were probably the seed for the formation of galaxy clusters.