Solutions

1st part

Exercise 1.1: Infrared radiation is divided into three ranges:

Near infrared (NIR):	780 nm - 5 µm
Mid-infrared (MIR)	5 – 30 µm
Far infrared (FIR):	30 – 600 µm

Key to terms:

$$1\text{ nm} = 1\text{ nanometre} = 0.000\,000\,001\text{ m} = 1 \times 10^{-9}\text{ m}$$

$$1\ \mu\text{m} = 1\text{ micrometre (micron)} = 0.000\,001\text{ m} = 1 \times 10^{-6}\text{ m}$$

(don't let that irritate you! The boundaries of the respective spectral ranges are not clearly defined. As a consequence, details may vary from one book to another).

Exercise 1.2: To a large extent, infrared light is blocked by the Earth's atmosphere, i.e. a proportion of it is absorbed as it makes its way through the atmosphere, thereby preventing it from reaching the Earth's surface. This is due primarily to the presence in the atmosphere of large volumes of water vapour. In addition, CO_2, ozone, methane, nitrous oxide and CFCs all have an important role to play. First and foremost, these molecules absorb radiation in the medium and far infrared range. Only the infrared radiation across isolated and very narrow wavelength ranges is able to penetrate unhindered as far as the Earth's surface. As a result, the best way of observing infrared light is through telescopes in outer space, well outside the Earth's atmosphere.

Exercise 1.3: Cold interstellar dust clouds, very distant from hot stars, have a temperature of about 15 to 20 K. Whenever they are located in the vicinity of hot stars, their temperatures can rise to between 30 and 50 K. The displacement law devised by the Austrian physicist Wien gives us a wavelength of λ_{max} for these temperatures at maximum radiation levels:

Temperature	λ_{max}
15 K	193.3 µm
20 K	145 µm
30 K	96,7 µm
50 K	58 µm

Interstellar dust clouds with temperatures of 15 to 20 K therefore achieve their maximum radiation levels across a range of 145 to 193.3 microns. In the case of dust clouds at temperatures of 30 to 50 K, the corresponding figures are 58 to 96.7 microns. Planck's radiation law states that cold interstellar dust clouds emit the majority of their light in the far infrared range.

Exercise 1.4: The redshift (z) indicates how much these wavelengths change when in outer space. The difference in outcome between a laboratory-based measurement of wavelength λ_0 (reference wavelength) and the same process conducted on a given star or galaxy with a wavelength of λ (Fig. 10) is referred to as the redshift, being parameter $z = (\lambda - \lambda_0) / \lambda_0$.

The underlying cause of this kind of wavelength shift is immaterial for definition purposes. If λ is greater than λ_0, radiation is being shifted towards larger wavelengths, i.e. towards the 'red end' of the electromagnetic spectrum. This is the explanation for the term 'redshift'. However, radiation can also shift towards shorter wavelengths, i.e. towards the 'blue end' of the electromagnetic spectrum. In this case, λ is smaller than λ_0 so this is referred to as a 'blueshift'.

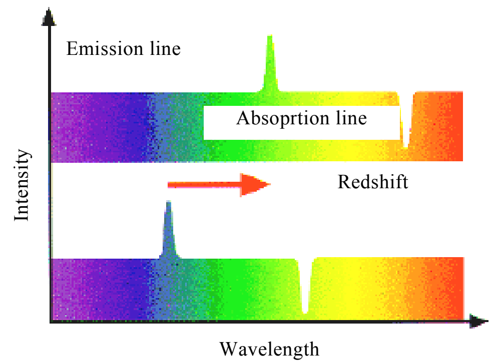

Fig. 10: Explanation of the redshift principle with the help of spectral lines: The spectral lines in the upper part of this image have shifted towards higher wavelengths, compared to the spectral lines in the lower part of this same image (source of image: [http://www.usm. uni-muenchen.de/people/ saglia/dm/galaxien/alldt/node50.html](http://www.usm.uni-muenchen.de/people/saglia/dm/galaxien/alldt/node50.html))

The redshift *z* becomes larger the further radiation is shifted towards the long-wavelength end of the electromagnetic spectrum. Whenever the values for *z* reach ≥ 1.0, this is referred to as 'high redshift objects'.

However, wavelength shifts not only occur in electromagnetic radiation – they also occur with sound waves.
A familiar example of this is the changing pitch of the sound of a siren as an ambulance drives past. The redshift and blueshift of sound waves in this example are caused by the Doppler effect.
The Doppler effect also gives rise to redshifts and blueshifts in astronomy, in response to the movement of stars or galaxies towards or away from us. However, whenever you are observing very distant objects such as high redshift galaxies, you are then usually only measuring redshifts. The redshift of these objects is a consequence of cosmic expansion (cosmological redshift) and has absolutely nothing to do with the Doppler effect. (regrettably, there are still cases where even some astronomers point to the Doppler effect when referring to cosmological redshifts. To do so is entirely incorrect). The expansion of the cosmos also stretches the radiation reaching us from distant objects. Due to the fact that light is also a participant in the process of cosmic expansion, the redshift effect is a direct cause of the expansion of the cosmos.

Exercise 1.5: Cold objects such as interstellar dust clouds transmit heat radiation at the far infrared end of the spectrum. The UV and visible light that we today receive from very distant objects is billions of years old and has undergone such an extreme redshift on its path through an expanding cosmos that we are now only able to observe it at the far infrared end of the electromagnetic spectrum. It therefore follows that, for this investigation, we need to measure far infrared light from cold radiation sources shrouded in dust as well as from high redshift radiation sources. Although these two phenomena are based on entirely different physical principles, we nonetheless need to employ the same tools to research them.

Exercise 1.6: The term 'early universe' is used to describe the first stage of development of our universe following the Big Bang. This also includes the time during which the first stars and galaxies formed. Due to the fact that these stars and galaxies are now high-redshift objects, i.e. objects whose radiation has shifted to the far infrared end of the spectral range, the Herschel space telescope is able to explore this 'early universe'. In layman's terms, that is "the place where high redshift objects dwell".

2nd part

Exercise 2.1: Herschel directs a surface area of $A = 4$ m x 7.5 m = 30 m^2 towards the Sun. Due to the fact that the space telescope is about 1% further from the Sun than the Earth, the level of solar radiation it receives is P, where:

$$P = 1370 \text{ W/m}^2 \; 30 \text{ m}^2 \; (1/1.01)^2 = 41926 \text{ W} = 42 \text{ kW}.$$

Exercise 2.2: The Earth radiates about 46 mW/cm^2 per cm^2 (256 K/300 K)^4 = 24.4 mW/cm^2. With the radius of the Earth R_{Earth} = 6371 km, this means that:

$$I_{total} = 4\pi \, R_{Earth}^2 \, P = 1.245 \; 10^{17} \text{ W}.$$

Black bodies: Idealised bodies which do not occur in Nature, which absorb all radiation directed at their surfaces, and which reflect it back. Due to their temperature, these bodies radiate heat precisely in accordance with Planck's law of radiation.

Albedo / radiation retrieval capability: This indicates what percentage of inbound radiation the surface of an object reflects back. An Albedo factor of 1.00 or 100 % indicates that the full extent of all inbound radiation is reflected back, while an Albedo factor of 0.00 or 0 % means that all inbound radiation is absorbed by the surface.

Absorption capability ε: The ability of a body to absorb radiation.

Why can a black object be used as a reference object for the Earth or for other objects which transmit heat radiation? With black bodies, it is known precisely how much radiation is emitted at a given wavelength, provided that the temperature of each body is known. It is therefore possible to make conclusions about the characteristics of real bodies for which the emission of heat radiation is dependent not on temperature but instead on material properties, size, expansion, etc.

The amount of radiation originating from Earth and reaching HERSCHEL is as follows:

$$I = (I_{total} \, F) \, / \, (4\pi \, r^2) = 1.245 \; 10^{17} \text{ W } 30 \text{ m}^2 \, / \, [4\pi \, (1.5 \; 10^9 \text{ m})^2] = 0.132 \text{ W}.$$

Exercise 2.3: The entire scope of radiation reaching Herschel is as follows: 42 kW + 0.132 W ≈ 42 kW. The radiation shield absorbs 5% of this, i.e. about 2100 W. In overall terms, this amount of radiation is reflected back into outer space, with about 5/6$^{\text{ths}}$ being returned towards the Sun and 1/6$^{\text{th}}$, or 350 W, being reflected back towards the device. It follows from this that, for each cm^2, the amount reflected back towards the Sun, P = 2100 W x (5/6) / (300 000 cm^2) = 5.83 mW. For temperature T on the radiation shield, it then follows that T = 379 K, because P = 5.83 mW = 0.05 x 46 mW x (T / 300 K)^4.

Exercise 2.4: The radiation from the primary mirror amounts to: $I_{mirror} = \pi \, r_{mirror}^2 \, \varepsilon \, \sigma \, (80 \text{ K})^4 = 17.9$ µW. The secondary mirror should have a surface area of approx. 1/300th of the primary mirror. At an estimated distance of about r_{mirror} = 1.7 m of the primary mirror, the emitted radiation would account for a proportion of about $(\pi \, r_{mirror}^2 / 300) / (0.5 \; 4 \, \pi \, r_{mirror}^2)$ = 1/600th striking the secondary mirror, i.e. 0.03 µW.

Exercise 2.5: Key to terms: T= 0.002898 mK/λ_{max}. For 60 microns, T= 48.3 K, for 600 microns, T= 4.83 K.

Exercise 2.6: When observing very cold objects on which the maximum λ_{max} of the intensity distribution of heat radiation (Planck curves) lies well within the infrared spectrum, the heat radiation of a 'hot' measuring device, i.e. one without any cooling applied, would be a major source of interference, and could conceivably even render the measuring process impossible. This would be the equivalent of trying to observe light from stars in the visible spectrum through an incandescent, white-hot telescope. The heat being radiated by such a telescope would simply completely overpower the weak starlight.

Exercise 2.7: On a Planck radiation curve, the ratios of intensity levels to different wavelengths are heavily dependent on temperature. If you establish the intensity of heat radiation emitted by a heavenly body at defined wavelengths, and if you are able to make a reasonable estimate of its emission capability, it is at least possible from the ratios of measured values of this kind to determine the approximate temperature of the heavenly body being observed.

Exercise 2.8: When it evaporates, 1 litre of liquid helium = 0.125 kg = 1/8 kg has absorbed the evaporation heat of 20.43 kJ / 8 = 2.554 kJ. To this, up to 15 K can be added, plus 0.6 kJ, up to 150 K a further 5.63 kJ and up to 200 K a further 7.38 kJ.

Exercise 2.9: The outer shell of Herchel is a cylinder with a radius $r = 2$ m and a height h =7.5 m. A cylinder of this kind has a surface area $A = 2 \pi r^2 + 2 \pi r h$ = 188.5 m^2. Its inward emission amounts to I = A ε σ (200 K)^4 = 17.13 mW. If we now consider the third radiation shield with a radius r = 1.9 m and height h = 7.3 m, we obtain an emission rating of I = 3.16 mW. For the innermost radiation shield, at a radius r = 1.5 m and height h = 7 m at a temperature of 15 K, a total emission level of I = 0.23 µW is obtained. It is possible for the innermost radiation shield and its emission level inwards to be even lower than this.

Exercise 2.10: Between 1 K and 4 K, the ability to conduct heat increases in a linear fashion as the temperature rises. In a temperature range of 2.8 K to 4 K, it is therefore possible to estimate the mean value for thermal conductivity as approximately 0.1 mW/(cm K). For heat conductivity between 4 K and 15 K, parameter $W($15 K) can be estimated as approx. 4 W/cm. If one end of the spoke has a temperature of 2.8 K and a temperature of 15 K at the other end after 1 m, the temperature of 4 K is achieved after covering a distance of z. The following then applies: The flow of heat from 0 m to z is as great as the flow of heat from z to 1 m:

$$Q = \kappa \text{ x } (T_2\text{-}T_1) \text{ x } A/1 = 0.1 \text{ [mW/(cm x K)] x (4 K-2.8 K) x (1 mm}^2/z) = 4 \text{ (W/cm) x [1 mm}^2/(1 \text{ m - } z)].$$

Shortening and reconfiguring this equation delivers this outcome:

0,0001x1.2 (1 m - *z*) = 4 - z => z x (4 – 0.00012) = 0.00012 m => *z* = 0.00003 m.

Therefore, for the calculation of thermal flow in this case, you only need to take the transport of heat from 4 K to *T* into account.

2.8K - 15K:	Q = 4	W/cm x 1 mm^2/lm = 0.0004 W,
2.8 K - 200 K:	Q = 545	W/cm x 1 mm^2/lm = 0.0545 W.

A doubling of the length halves the thermal flow. Tripling the cross sectional surface area causes the thermal flow rate to treble.

Exercise 2.11: You reach a temperature of 1.6 K by releasing helium into space through a feedback control valve until the pressure of the helium only measures 4.8 hPa.

Exercise 2.12: The vapour pressure must be 0.009 hPa.

Exercise 2.13: To achieve a thermal power rating of 10 µW over a 45 hour period, you need energy of 1.62 J. For this, you need 1.62 J / 8302 J/kg = 0.1951 g ^{3}He.

Exercise 2.14: During every measurement of an astronomical object, the signal from the measured object has interference levels superimposed which arise in the detector and in the associated electronics: a hissing or crackling noise. Since this background noises occurs regardless of the object being measured, you first measure the object, then an area of the heavens without that object. Then you subtract the measuring signal without object from the measuring signal with object. This makes it possible to reduce the influence of this background noise on the measurement to a very significant extent.