Fluid static pressure: Exercise

Consider a container filled with a fluid and apply an acceleration a on it, towards the upper side. Show that pressure changes following the law: $p = p_o + \rho h (g + a)$.

If a is directed in the opposite direction, so that $p = p_o + \rho h (g - a)$, what happens in an ideal weightless environment?

Solution:

Bear in mind the forces equation of a fluid element: acceleration, pressure on the upper surface, pressure on the bottom, weight of the element.)

y ↓

(surface y = 0)

$\Sigma F = ma = -\rho Adya$
$-\rho Adya = +pA -(p+dp)A + \rho gAdy$
m = mass of the fluid element
A = surface of the fluid element
dy = depth of the fluid element
p = pressure
dp = increment of pressure in dy
$-\rho Adya = -dpA + \rho gAdy$
$dp/dy = +\rho (g+a)$
$\int dp = \int + \rho(g+a)dy$
between 0 and h.

If p_0 is the pressure on the surface of the liquid (y=0) and p is the pressure corresponding at depth=h, then
$p= p_o + \rho h (g+a)$
If the acceleration is directed in the opposite direction
$\rho Adya = +pA - (p+dp)A + \rho gAdy$
and $p = \rho h (g-a)$
then in a weightlessness environment g = a and the fluid static pressure is equal to 0.