Convection: Exercise

In describing convection, the Rayleigh number (Ra) is often introduced. The Rayleigh number is a dimensionless quantity that expresses the balance between effects that promote or suppress convection. It depends on the characteristics of the fluid, of the container and on the temperature difference applied. In a horizontal fluid layer, convection occurs if the Rayleigh number exceeds a critical value of 1708.

$Ra = (\alpha\ g\ \Delta T\ L^3)/(\nu\chi)$

α = thermal expansion coefficient
g = acceleration due to gravity
ΔT = temperature difference across the fluid (positive if heated from below)
L = layer thickness
ν = kinematic viscosity
χ = thermal diffusivity

On the ground, we want to observe convection in a 1 cm deep layer of water by heating it from below. Knowing that $\alpha = 2 \times 10^{-4}\ K^{-1}$, $\nu = 0.01\ cm^2/s$ and $\chi = 1.5 \times 10^{-3}\ cm^2/s$, calculate the minimum temperature difference needed to trigger convection.

Let us suppose we want to repeat the experience on board the ISS. What is now the minimum temperature difference if $g_{ISS} = 10^{-6} g$?

That is the reason why on board the ISS, thermal convection must be forced and does not occur spontaneously.

What happens in a situation of ideal weightlessness?

Solution:

$Ra \geq 1708$, $\Delta T_{earth} \geq (1708\ \nu\chi)/(\alpha\ g\ L^3) = 1.3\ K$

$\Delta T_{ISS} \geq (1708\ \nu\chi)/(\alpha\ g\ L^3) = \Delta T_{earth}\ (g/g_{ISS}) = \Delta T_{earth}\ 10^6\ K = 1.3\ 10^6\ K$

Ra = 0 Convection does not occur