

Combining the midlatitudinal and equatorial background error in global data assimilation

Heiner Körnich and Erland Källén

Department of Meteorology,
University of Stockholm

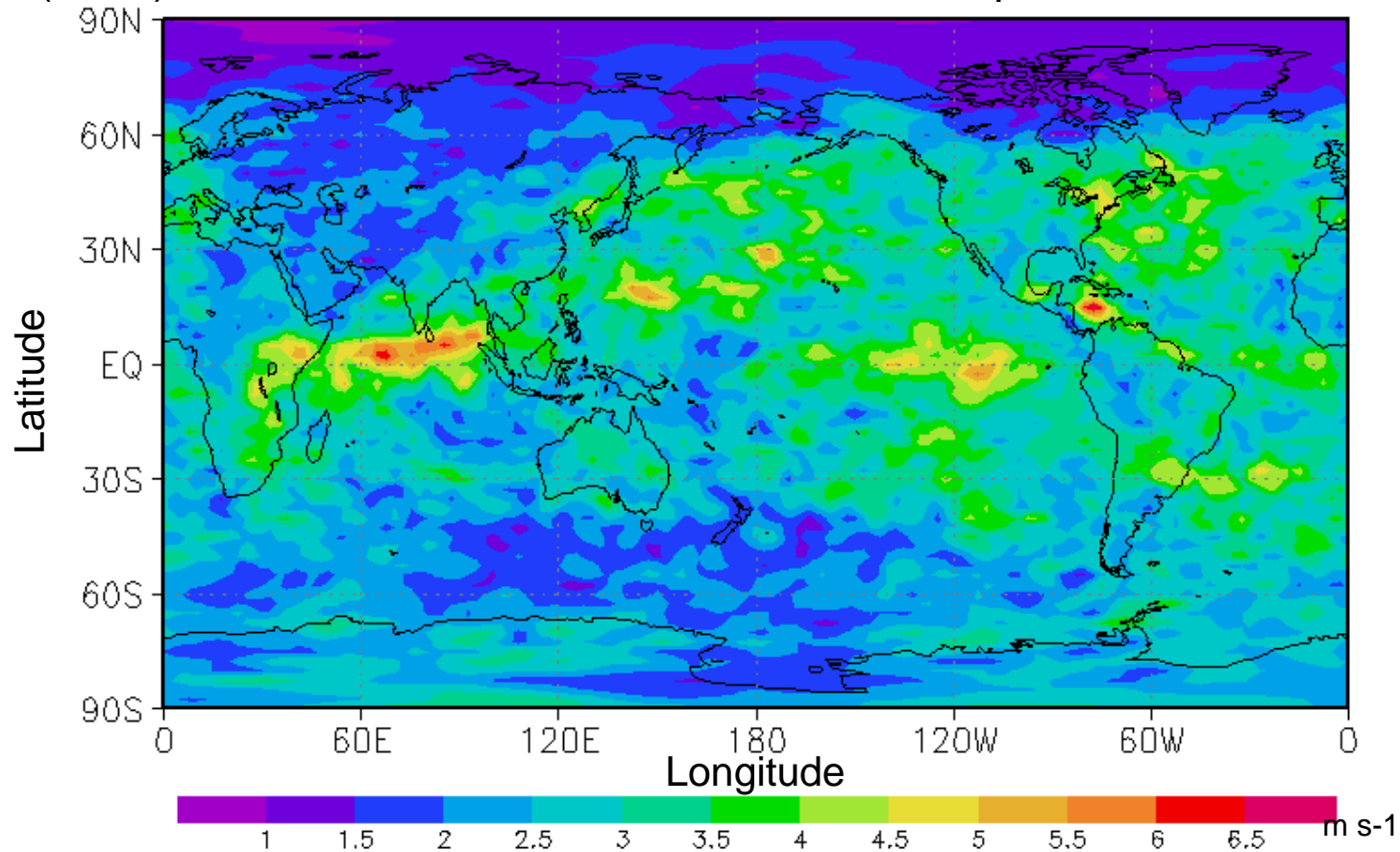
ADM-Workshop 26 September 2006

Overview

- Problem: Global data assimilation contains no tropical mass/wind relationship
- Methods: Background error in data assimilation via Hough modes
- Results:
 - Response to single observations
 - Effect of ADM-data in global data assim.
- Summary

The need of tropical wind data

Standard deviation of 48-hour forecast errors in the zonal wind field (m s^{-1}) of the ECMWF model at 200 hPa for September 2005.



➔ Line-of-sight wind by ADM-Aeolus

Variational data assimilation

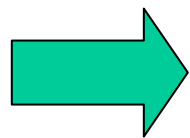
$$J(\delta\mathbf{x}) = \underbrace{\delta\mathbf{x}^T \mathbf{B}^{-1} \delta\mathbf{x}}_{\text{Background}} + \underbrace{(\mathbf{y} - H[\mathbf{x}_b + \delta\mathbf{x}])^T \mathbf{R}^{-1} (\mathbf{y} - H[\mathbf{x}_b + \delta\mathbf{x}])}_{\text{Observational error}}$$

$\delta\mathbf{x}$: Increment \mathbf{x}_b : Background \mathbf{y} : Observations

\mathbf{R} : Observation error covariance

\mathbf{B} : Background error covariance

H : Forward model

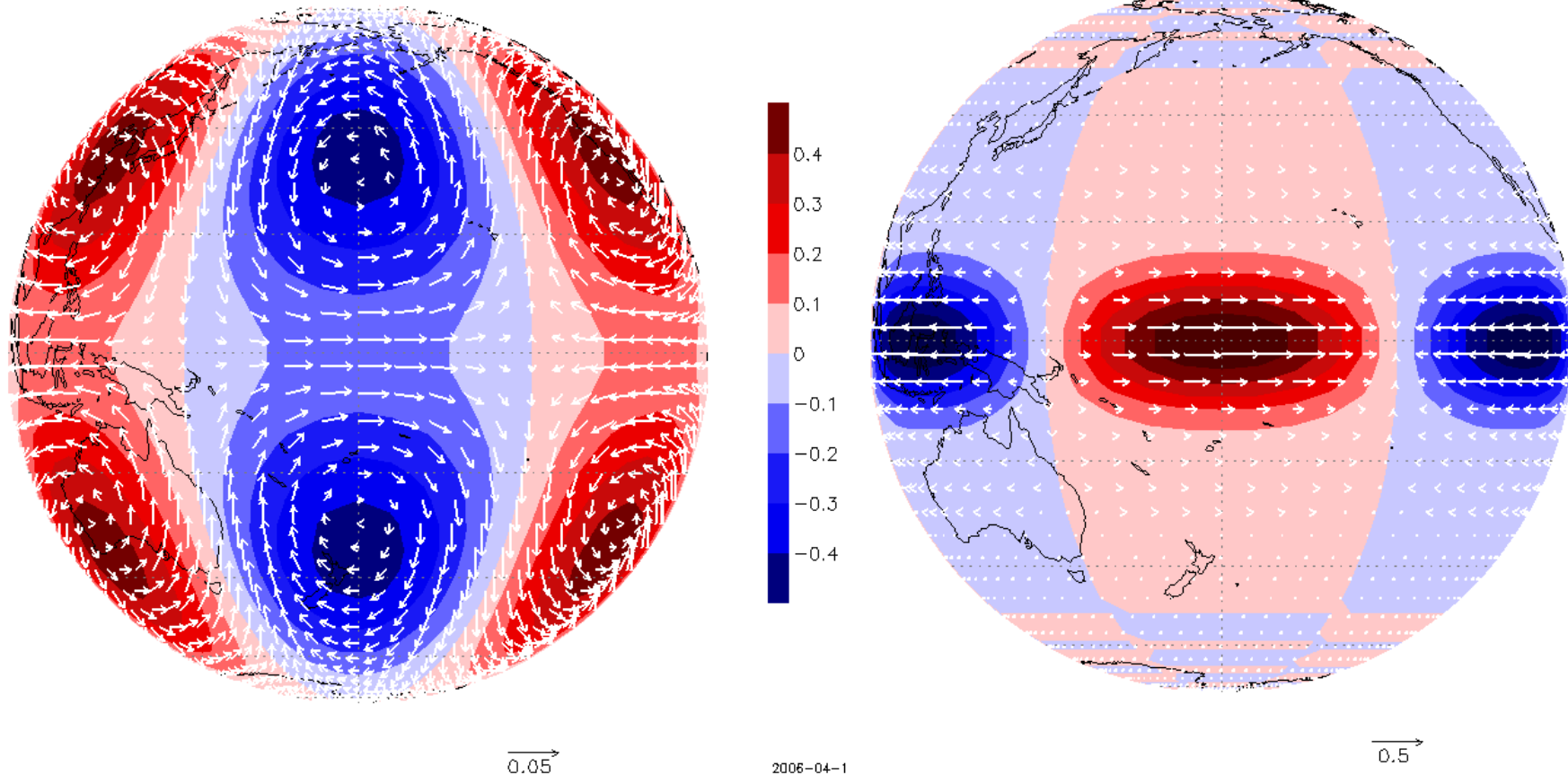


This study: global shallow water model (2d)

Midlatitudinal and equatorial Hough modes

Rossby mode, $n=1$, $m=3$, $H=10^4$ m

Kelvin mode, $m=3$, $H=25$ m



Hough modes with zonal wave number $m=3$.

Shadings show geopotential height (m), vectors show winds (m/s).

Problem: Modes not orthogonal. Vertical structure function ?

A priori knowledge of the background errors (**B**)

- Midlatitudes:

- Geostrophy
- Rossby waves
- mixed Rossby-gravity w.

→ Hough modes with equivalent depth

=10.000 m

- Equator:

- Kelvin waves
- Equatorial Rossby waves
- Mixed Rossby-gravity w.
- "Slow" west- and eastward inertia-gravity w.

→ Hough modes with equivalent depth=25 m

**Selection
+
Weighting**

For the assimilation each mode is assigned a certain variance which is deduced eg. from forecast errors.

FOR NOW: Red spectrum for each mode is assumed.

Combining midlatitudinal and equatorial \mathbf{B}

-> Now, the increment is split in two: $\delta\mathbf{x} = \delta\mathbf{x}_m + \delta\mathbf{x}_e$

-> Minimize cost function via control variables :

$$\chi_m = \mathbf{L}_m \delta\mathbf{x}_m$$

$$\chi_e = \mathbf{L}_e \delta\mathbf{x}_e$$

$\mathbf{L}_{m/e} = \mathbf{D}_{m/e} \mathbf{M}_{m/e}$ with $\mathbf{D}_{m/e}$: Normalisation with standard deviation of each Hough mode

$\mathbf{M}_{m/e}$: Projection on Hough modes

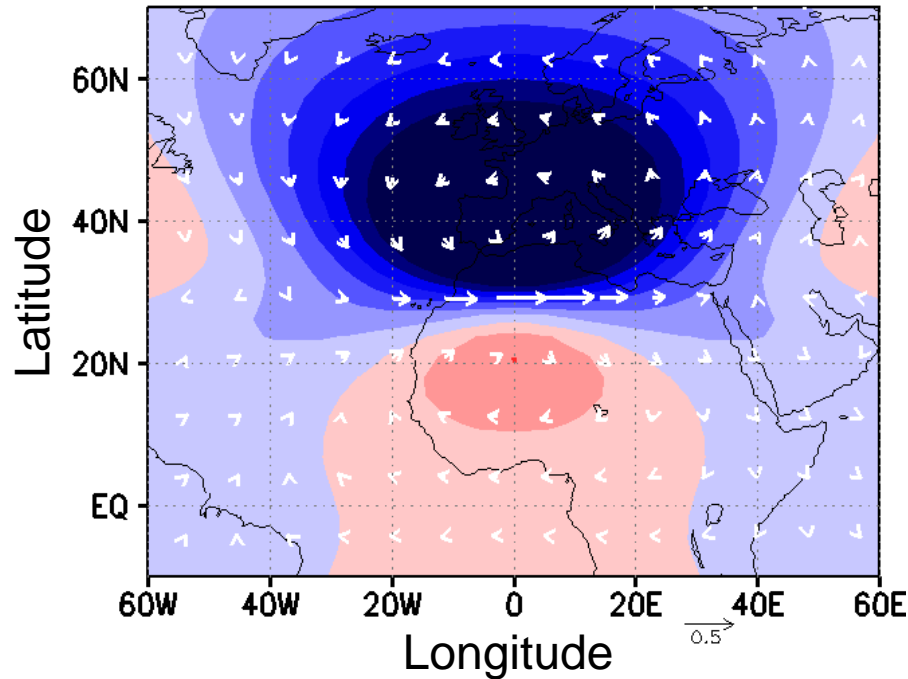
$$J(\chi_m, \chi_e) = \chi_m^T \chi_m + \chi_e^T \chi_e +$$

$$+ (\mathbf{y} - H[\mathbf{x}_b + \mathbf{L}_m^{-1} \chi_m + \mathbf{L}_e^{-1} \chi_e])^T \mathbf{R}^{-1} \dots$$

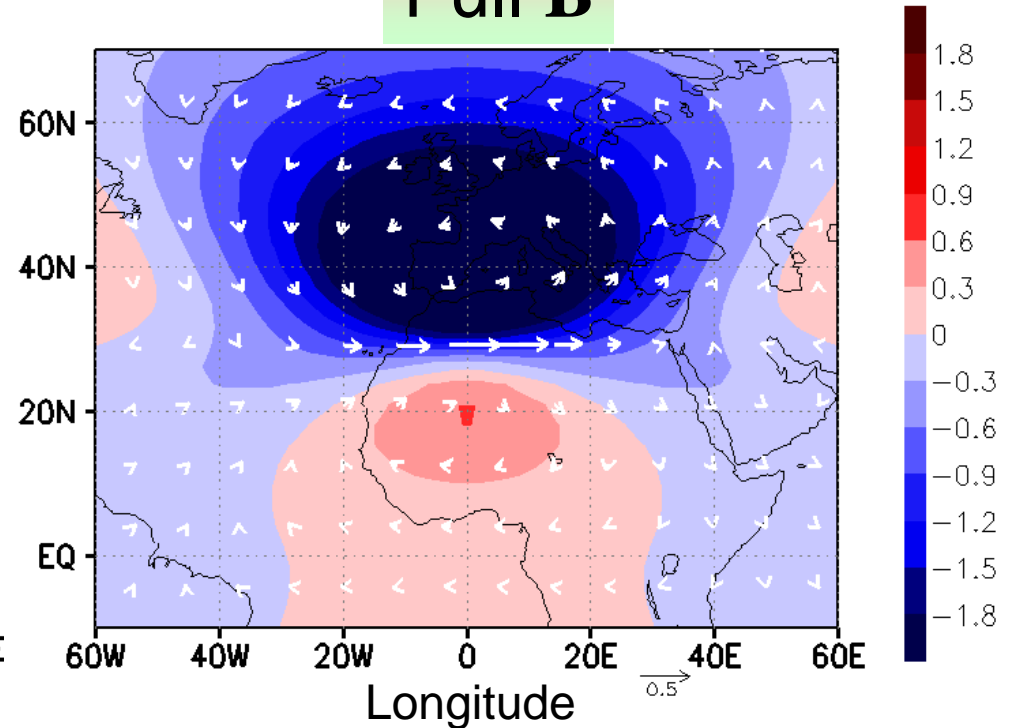
$$\dots (\mathbf{y} - H[\mathbf{x}_b + \mathbf{L}_m^{-1} \chi_m + \mathbf{L}_e^{-1} \chi_e])$$

Single zonal wind observation in midlatitudes

Only midlat. **B**

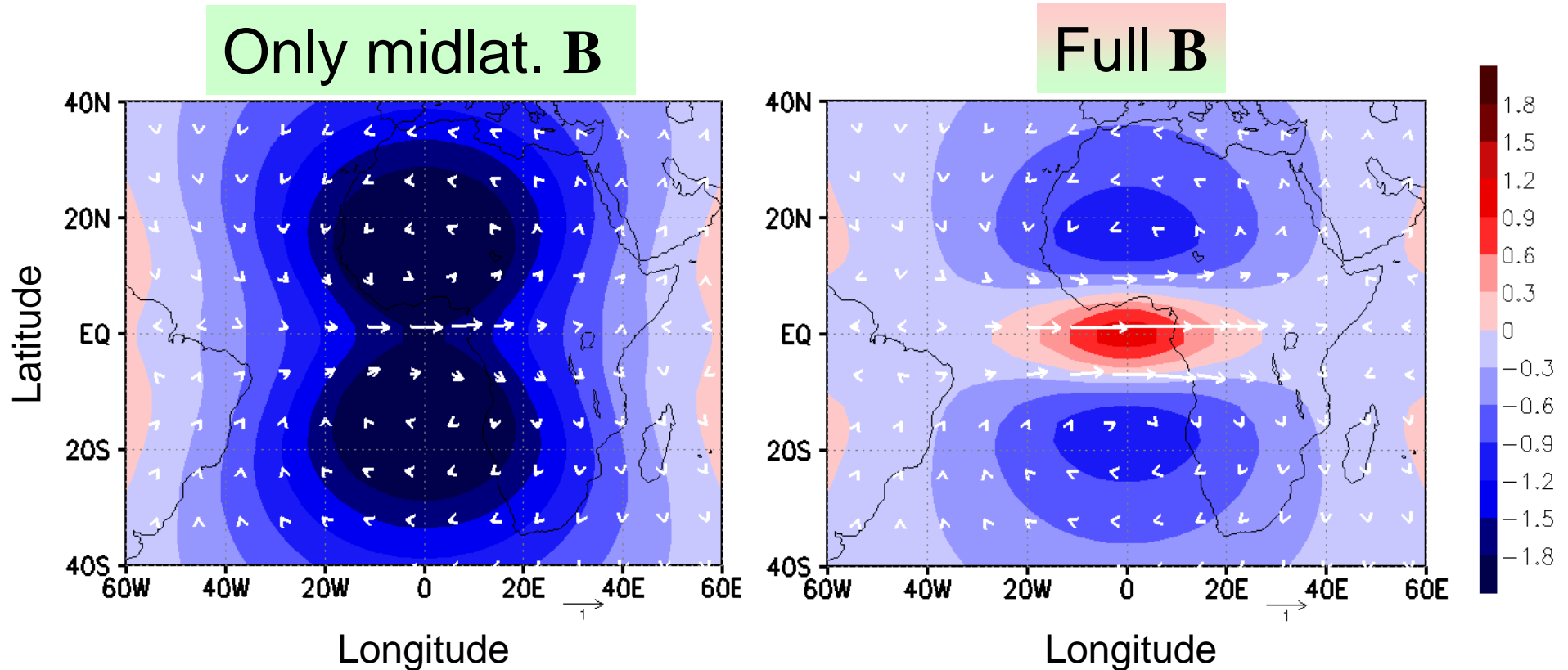


Full **B**



Analysis increment to a single zonal wind observation in midlatitudes at 30°N 0°E. Shadings show geopotential height (m) with positive values red and negative blue. Arrows show wind (m/s).

Single zonal wind observation at the equator



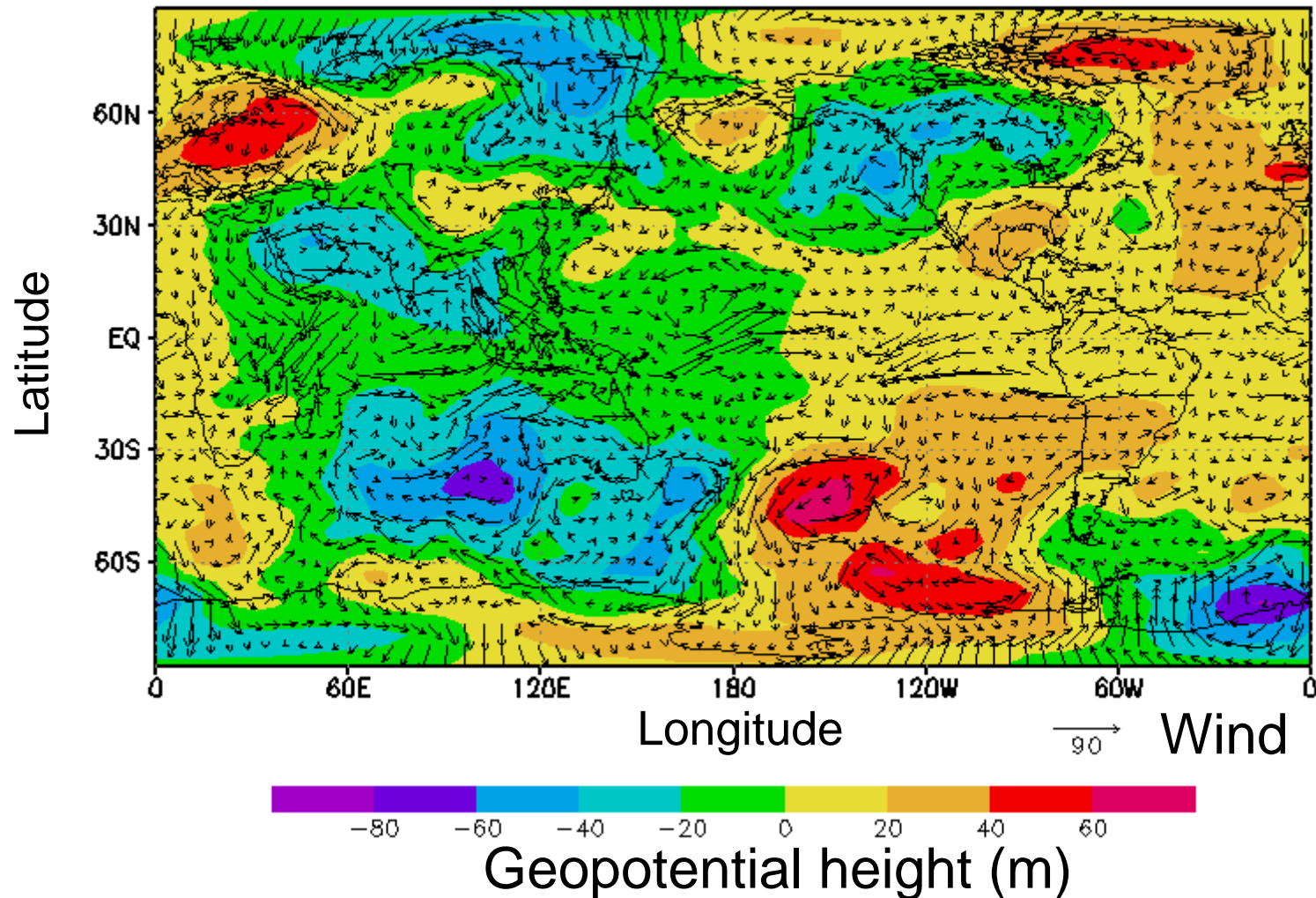
Analysis increment to a single zonal wind observation in midlatitudes at $0^{\circ}\text{N } 0^{\circ}\text{E}$. Shadings show geopotential height (m) with positive values red and negative blue. Arrows show wind (m/s).

Application to global data: Simulation experiments

1. Create **artificial data** set with known covariances for eq. and midlat. Modes.
2. **Reconstruct** background error covariances **B** (ie. standard deviation of each mode $\mathbf{D}_{m/e}$).
Problem: Non-orthogonality of modes
3. Create **observations** with random errors.
4. **Data assimilation** experiments
 - Influence of ADM-wind data on analysis
 - Effect of "wrong" or "right" background error

Artificial data

Random data x_r , day 1

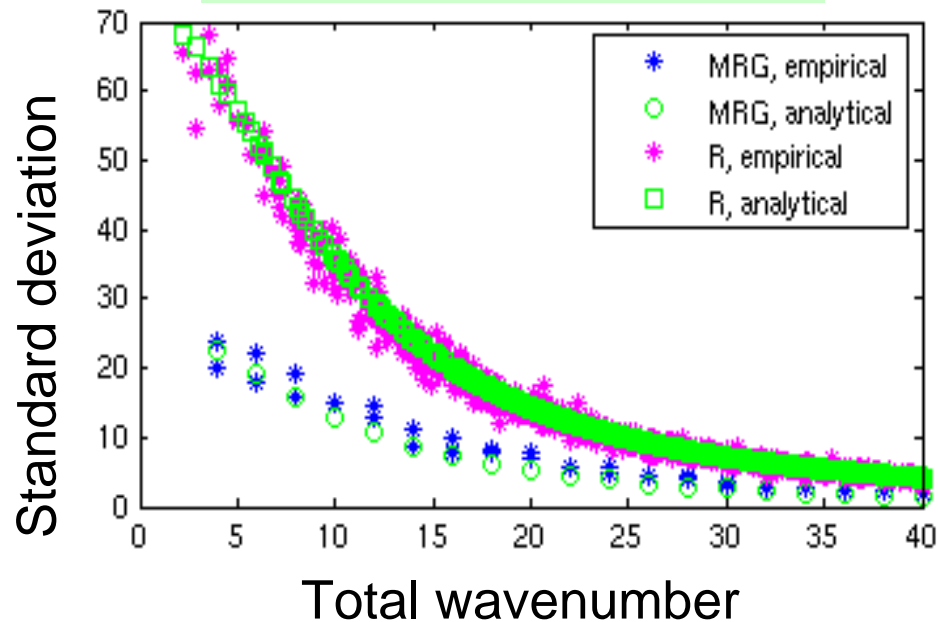


Reconstructing B

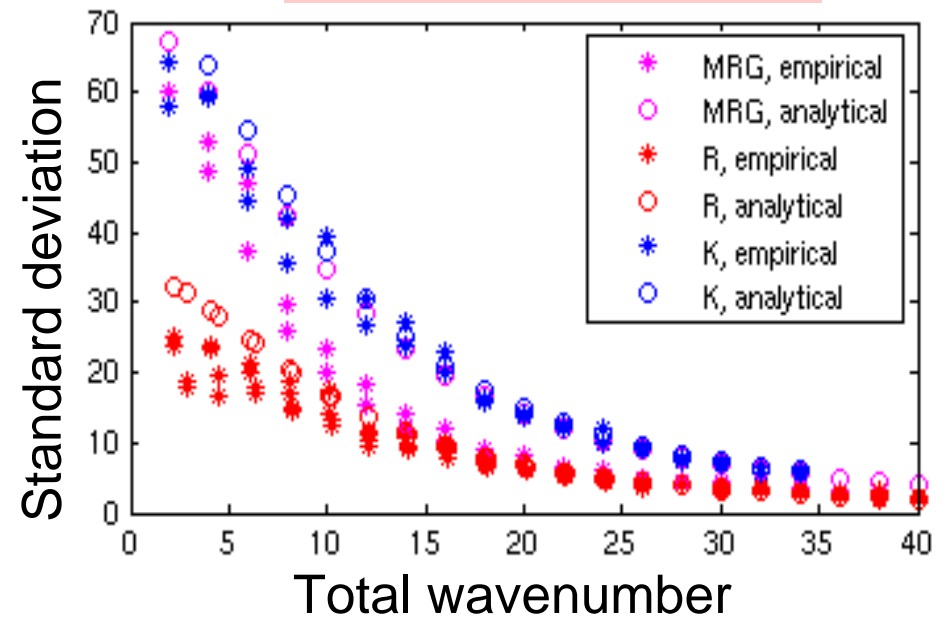
- Determine daily amplitude $\mathbf{a}_{m/e}$ of modes by minimizing error:

$$\left\| \mathbf{x}_r - \mathbf{a}_m \mathbf{M}_m - \mathbf{a}_e \mathbf{M}_e \right\|^2$$

Midlatitudinal modes

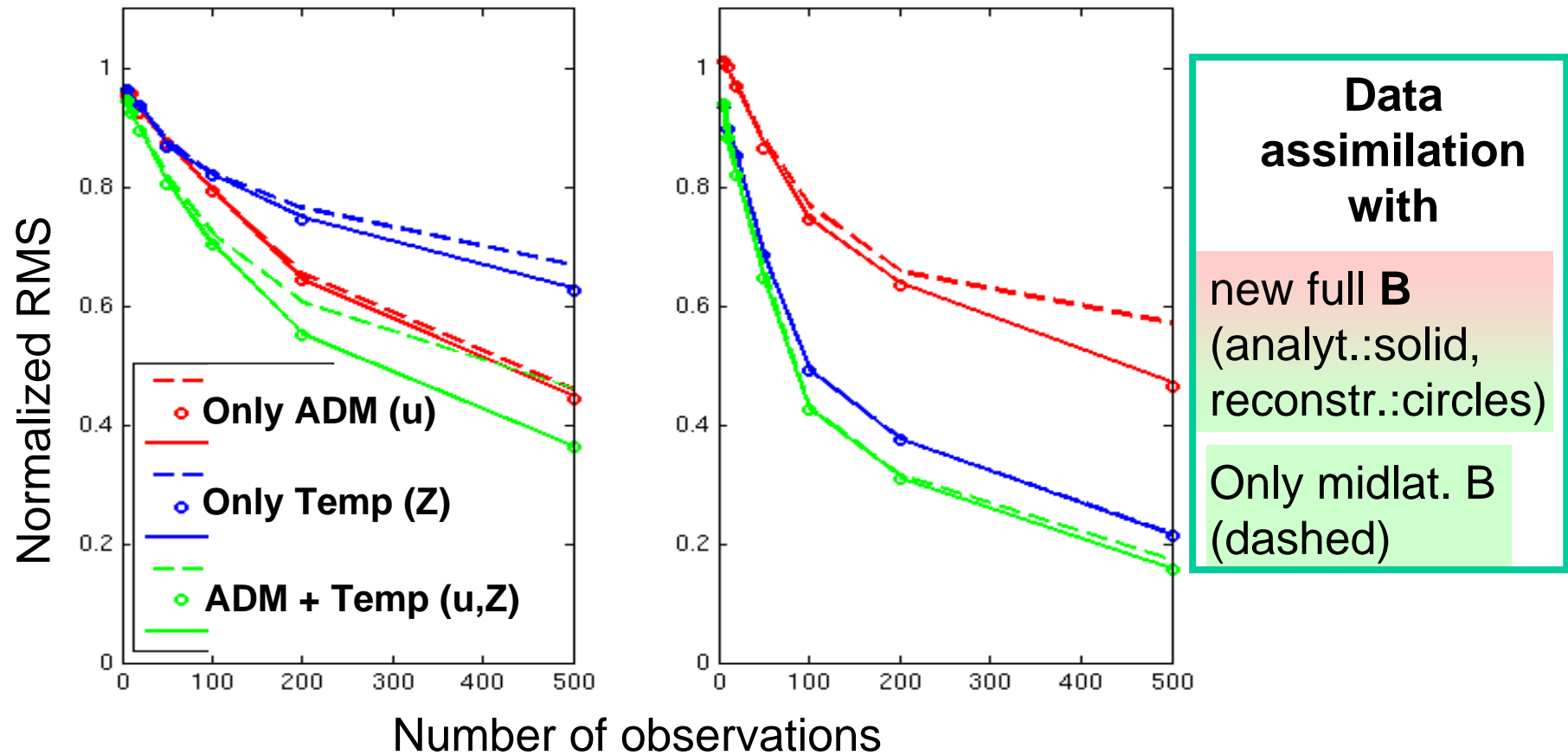


Equatorial modes



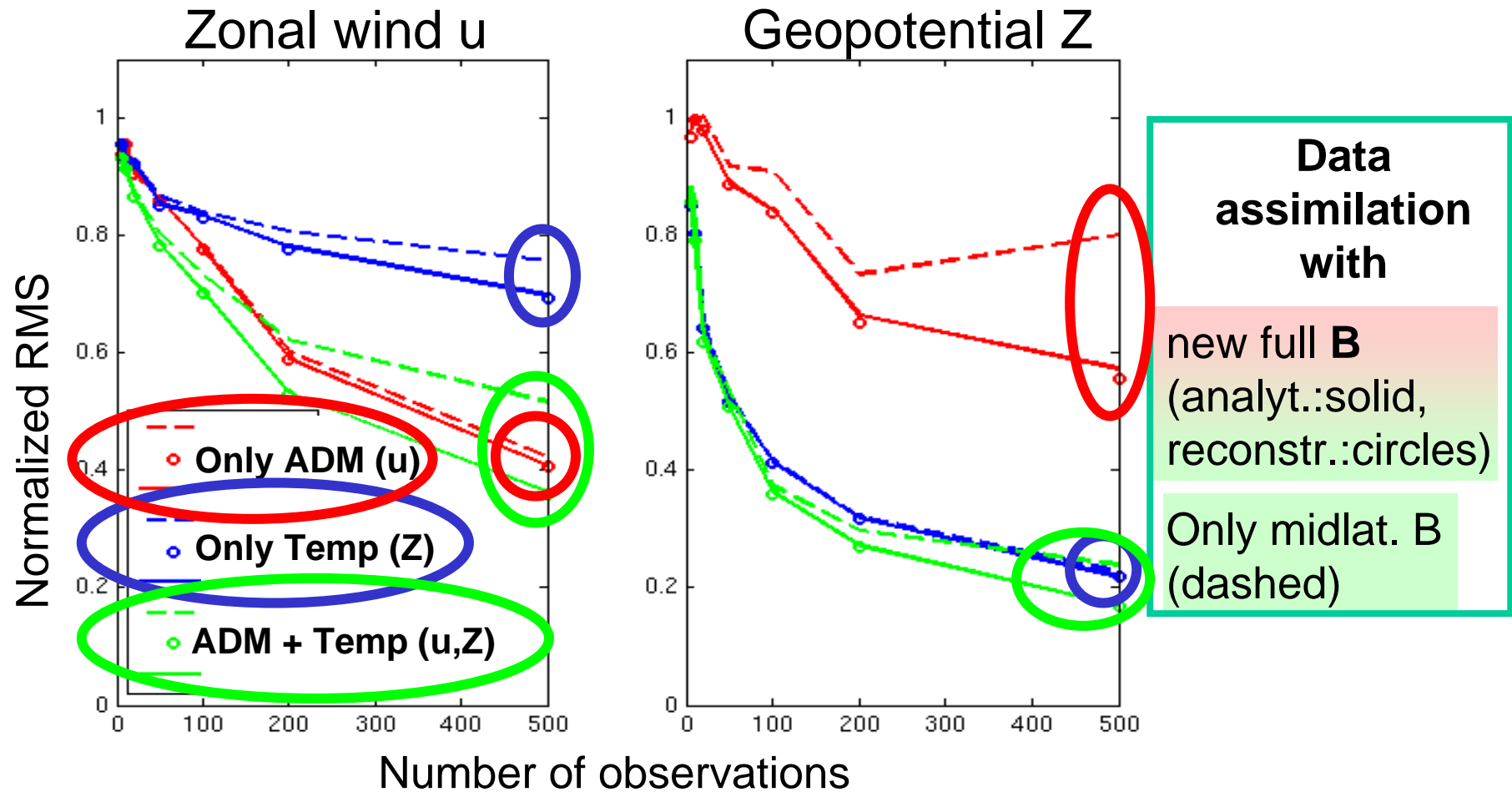
Effect of ADM-wind data in the global perspective

Global Root Mean Square Error of
Zonal wind u Geopotential Z



Effect of ADM-wind data in the tropical perspective

Equatorial (30°S-30°N) Root Mean Square Error of



Summary

- A combined background error formulation includes the equatorial waves following Žagar (2004) retaining the midlatitudinal approach.
- A minimization technique retrieves the background error covariances for the non-orthogonal set of Hough modes.
- The ADM-Aeolus wind data will improve the global analysis with the new background error formulation, especially by reducing the tropical error.